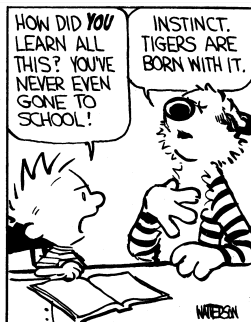
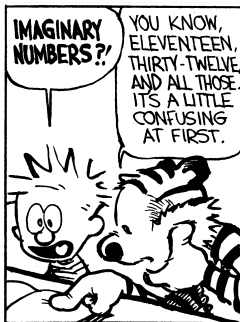
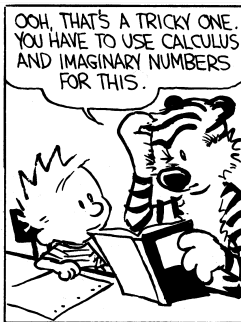
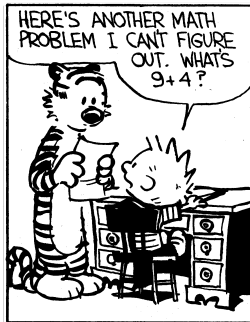


# The Impact of Computer Science on Mathematics Education



Zwei Probleme der inhomogenen diophantischen  
Approximation



Jürgen Bokowski

1973



Obere Schranke zur Gitterpunktsanzahl konvexer Körper

Jürgen Richter-Gebert

1992



On the Realizability Problem of Combinatorial geometries-  
Decision Methods

Ulrich Kortenkamp

1999



Foundations of Dynamic Geometry

er Charakteristiken

1970 1980 1990 2000 2010 2020

Jahr der Dissertation

# Interactive Geometry

## Objects

Points



Lines



Circles, Conics

Numbers

...



## Relations

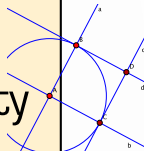
incidence

orthogonality

parallelism

is x-coordinate

...



**Easy, isn't it?**

**No.**



**Demo**

**Easy, isn't it?**

**It's even ~~now~~ worse.**

**Demo**

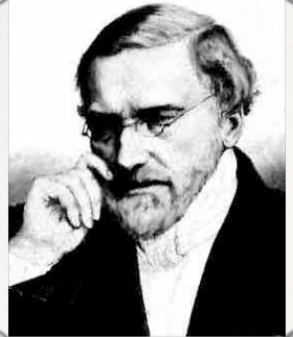
**Solution:**  
**Not easy, but nice.**

# What we want

---

## **Continuity Principle** by Poncelet

- ▶ “A relation known to hold with sufficient generality for a given figure also holds for all other figures that may be derived from it by continuous variation”



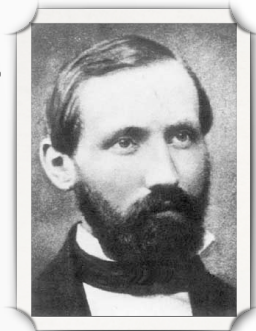
Victor Poncelet

Small changes in input lead to  
small changes in output

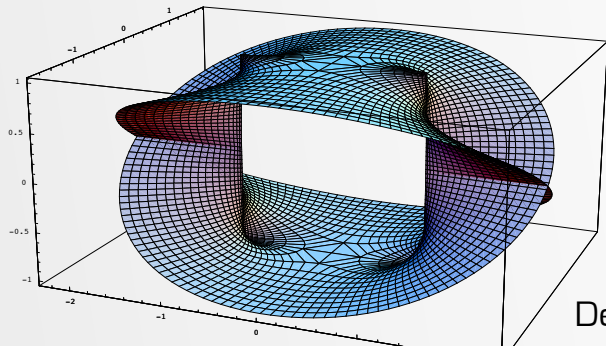
# What we have to do

---

- ▶ Work with complex homogeneous coordinates
- ▶ Do analytic continuations on a Riemann surface




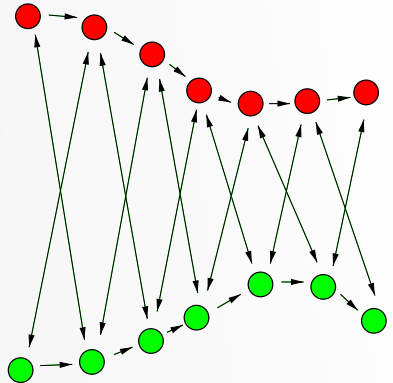
Bernhard Riemann



Details: Broser 2008

# Underlying algorithm

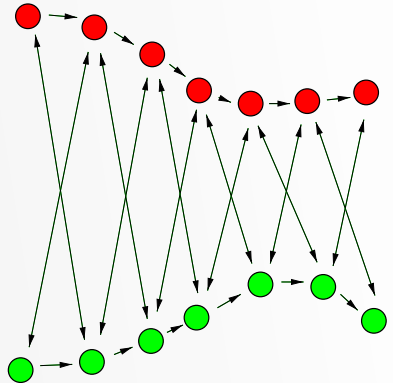
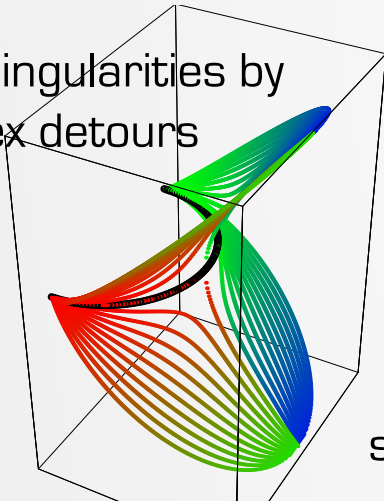
- ▶ Use “close-to” tracing algorithm
  - ▶ Avoid singularities by complex detours
- 



# Underlying algorithm

---

- ▶ Use “close-to” tracing algorithm
- ▶ Avoid singularities by complex detours



see Video on Vimeo



# Consequences

---

- ▶ All dependent elements behave continuously (in fact analytically)
- ▶ We have a notion of geometric theorem
- ▶ We can check theorems numerically
- ▶ Everything works “as expected” (for mathematicians, which is not what most people expect)



Felix Klein

aturali – De Absoluta Dei Simplicitate,  
id est Rempublicam In Microcosmo Conspiciam

ann Christoph Wichmannshausen  
1735



Disputationem Moralem De Divitiis Secundum Jus Naturae

Christian Hausen  
1713



De corpore scissuris figurisque non cruetando ductu

Abraham Gotthelf Kästner  
1739



Theoria radicum in aequationibus

Johann Friedrich Pfaff  
1786



Commentatio de orbitis et occasibus s  
classicos commemoratis

Carl Friedrich Gauß  
1799



Demonstratio nova th  
algebraicam rationale  
reales primi vel secun

Christian Ludwig G  
1812



Method  
parallac  
die

Julius  
1823

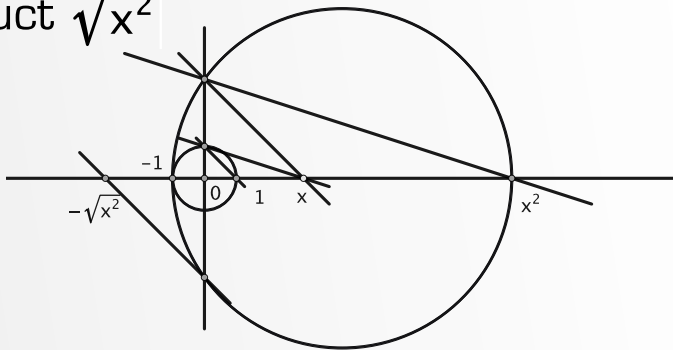


1690 1700 1710 1720 1730 1740 1750 1760 1770 1780 1790 1800 1810 1820 1830

# Functions and Roots

---

- ▶ von-Staudt-Constructions: add & multiply  
(field structure in Projective Geometry)
- ▶ Euclid: find square root
- ▶ we can construct  $\sqrt{x^2}$



# Similar: Oriented Areas

---

- ▶ Talk of Thomas Banchoff
- ▶ Absolute value function is not analytic
- ▶ If we need a notion of "Theorem" we better avoid it
- ▶ If we allowed it, we could construct

$$\frac{|x|}{x} = \begin{cases} 1 & \text{for } x > 0 \\ -1 & \text{for } x < 0 \end{cases}$$



Daß es mehrere Winkelbegriffe gibt, ist schon früher zur Sprache gekommen. Manche Didaktiker wollen uns davon überzeugen, daß nur einer der richtige sei. Ordnungsliebe ist lobenswert, aber sie sollte nicht so weit gehen, daß man wichtige Begriffe verbietet, weil sie nicht ins System passen.

Mathematik als pädagogische Aufgabe,  
Hans Freudenthal, Klett 1973

It was mentioned earlier that there are several notions of angle. Some educationists want to convince us that only one of them is the correct one.

Tidiness is wonderful, but it should not be exaggerated if this leads to banning important notions just because they don't fit into the system.

Mathematik als pädagogische Aufgabe,  
Hans Freudenthal, Klett 1973

# Concepts & Processes

## ► Change

In every meaning of the word!

Wonderful Example:

Hyperbolization of Ornaments  
(von Gagern & Richter-Gebert)

► [http://www.combinatorics.org/Volume\\_16/PDF/v16i2r12.pdf](http://www.combinatorics.org/Volume_16/PDF/v16i2r12.pdf)  
needs Hypergeometric Functions (Klein!)

Erster Teil.

Die geschichtliche Entwicklung bis  
einschließlich RIEMANN'S Arbeit aus  
dem Jahre 1857 (\*).

Einleitung:

Erstes Auftreten der hypergeometrischen  
Funktion: Reihe, Differentialgleichung,  
bestimmtes Integral.

Im Mittelpunkt unserer Betrachtungen über die hypergeometrische Funktion wird die Arbeit von RIEMANN stehen: „Beiträge zur Theorie der durch die GAUßSCHE Reihe  $F(a, b; c; x)$  darstellbaren Funktionen.“ Abh. d. Kgl. Ges. d. W. z. Berl. Bd. 7, 1857 (= RIEMANN [1], S. 67ff.) (\*\*).

Das vollständige und allseitige Verständnis dieser Arbeit und ihrer Tragweite zu erwecken, wird ein Hauptziel meiner Vorlesung sein.

Übrigens schließe ich mich zunächst an die geschichtliche Entwicklung unseres Gegenstandes an.

Es sind drei koordinierte Gesichtspunkte, unter welchen sich, geschichtlich betrachtet, den Mathematikern die hypergeometrische Funktion zuerst dargeboten hat:

1. als Potenzreihe: *hypergeometrische Reihe*,
2. als Lösung einer gewissen linearen Differentialgleichung 2. Ordnung: *hypergeometrische Differentialgleichung*,
3. als bestimmtes Integral: *hypergeometrische Integrale*.

Alle diese Gesichtspunkte treten bereits bei EULER hervor.  
Zu 1. Unter der *hypergeometrischen Reihe*, auch als *gewöhnliche hypergeometrische Reihe* oder *GAUßSCHE Reihe* bezeichnet, versteht man folgende Potenzreihe:

$$F(a, b; c; x) = 1 + \frac{a \cdot b}{1 \cdot 2} x + \frac{a(a+1) \cdot b(b+1)}{1 \cdot 2 \cdot c(c+1)} x^2 + \dots + \frac{a(a+1)(a+2) \cdot b(b+1)(b+2)}{1 \cdot 2 \cdot 3 \cdot c(c+1)(c+2)} x^4 + \dots$$

Dabei sollen  $a, b, c$  (zunächst reelle) Zahlen bedeuten, und  $c$  darf weder Null noch eine negative ganze Zahl sein.

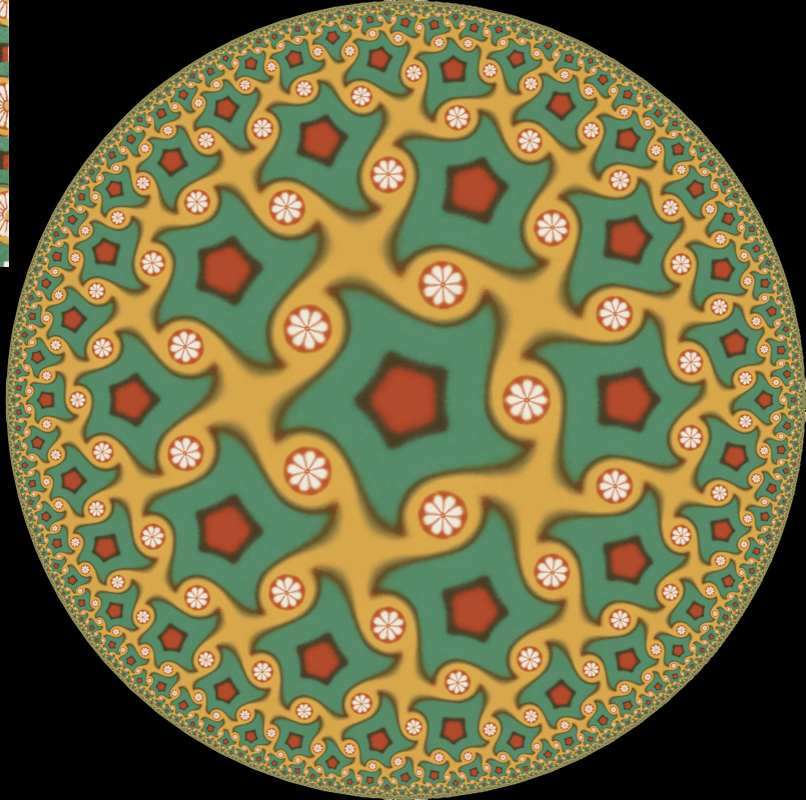




by von Gagern &  
Richter-Gebert



by von Gagern &  
Richter-Gebert

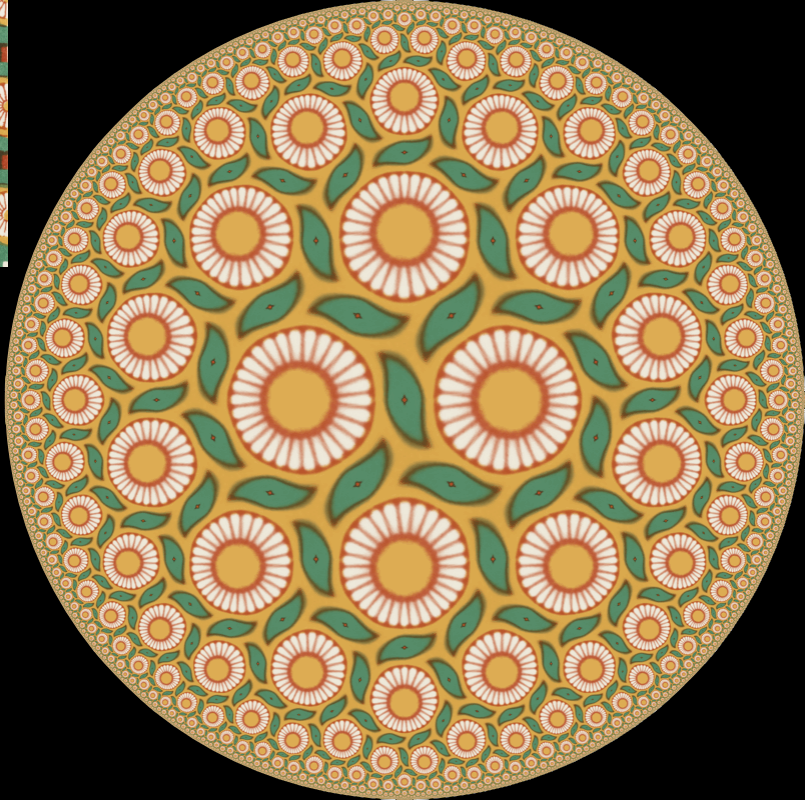


by von Gagern &  
Richter-Gebert

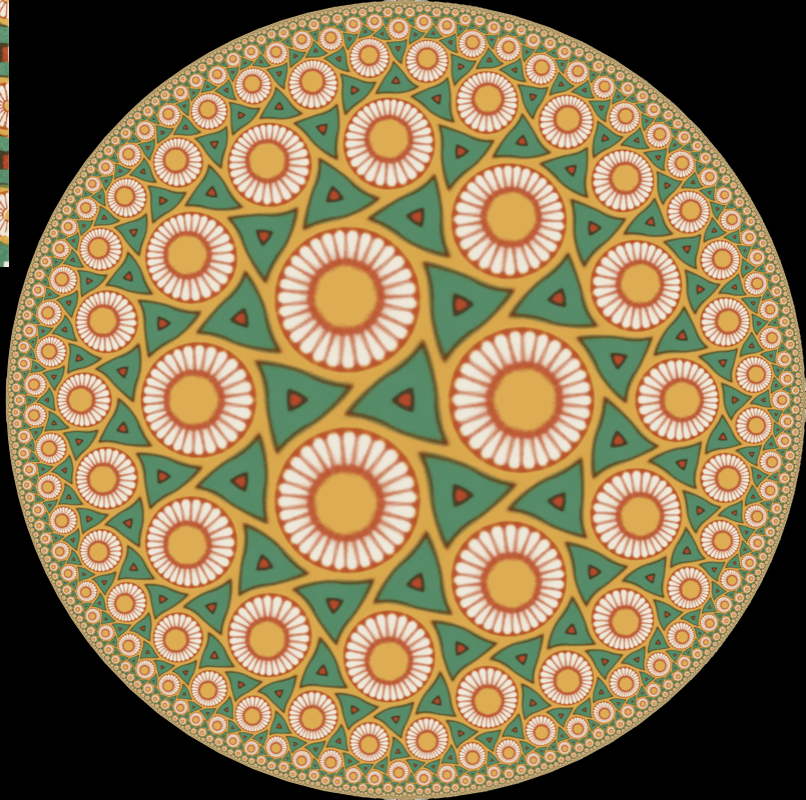


by von Gagern &  
Richter-Gebert

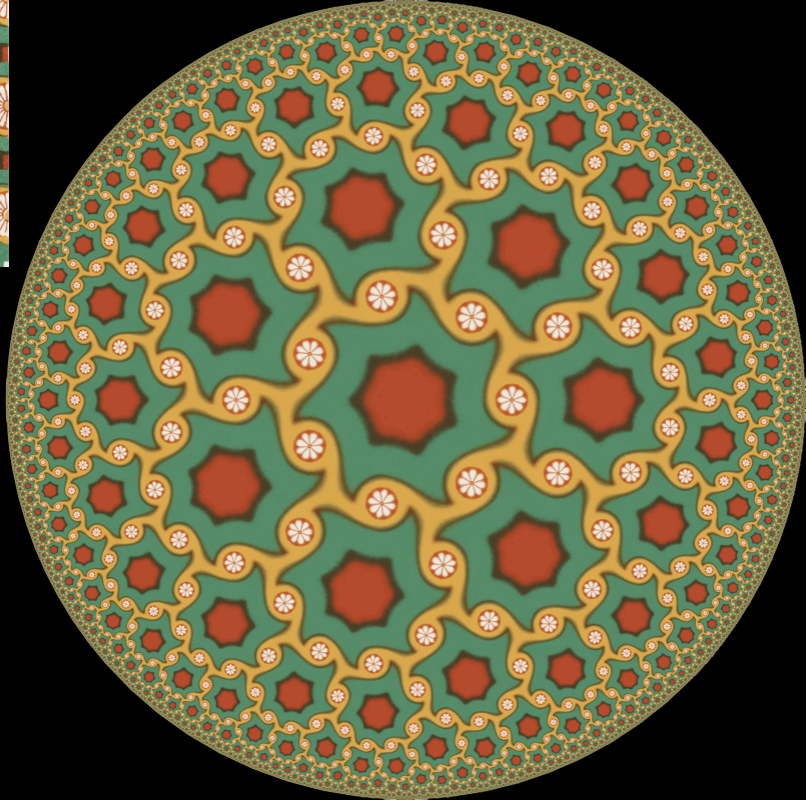




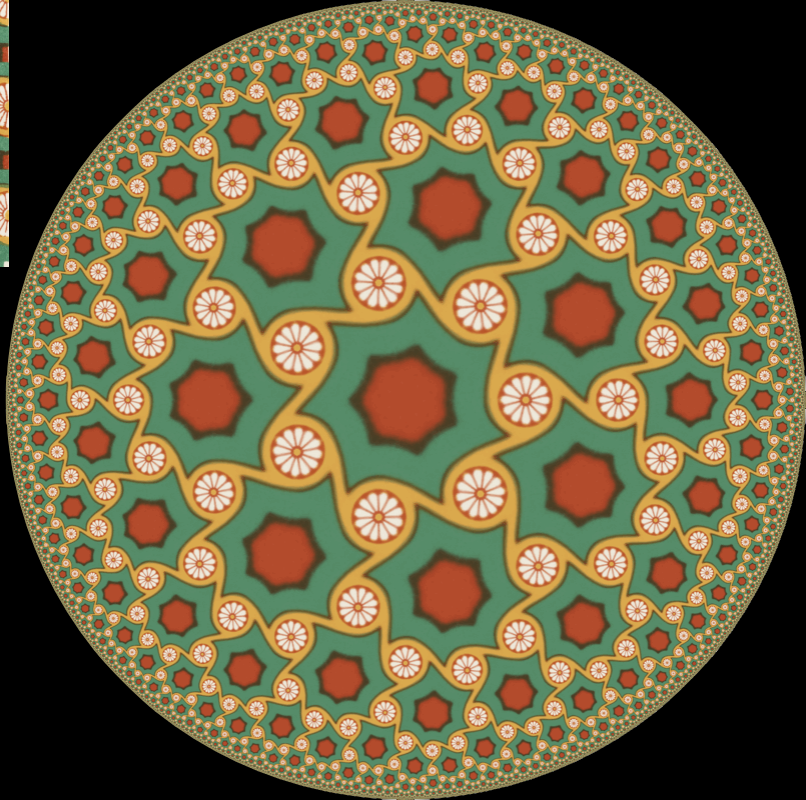
by von Gagern &  
Richter-Gebert



by von Gagern &  
Richter-Gebert



by von Gagern &  
Richter-Gebert

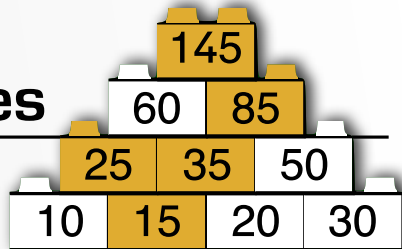


by von Gagern &  
Richter-Gebert



# Concepts & Processes

---



## ► Change

In every meaning of the word!

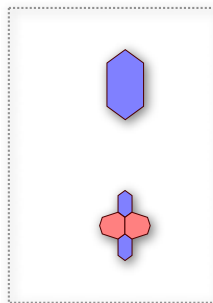
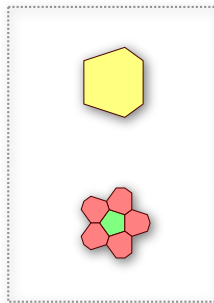
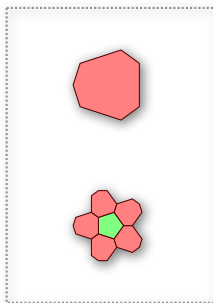
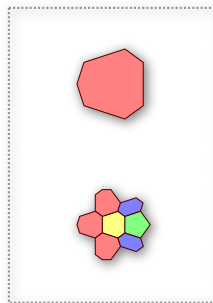
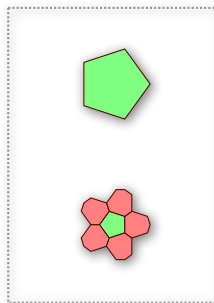
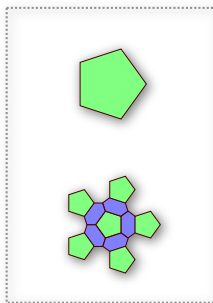
On every level (variation, metavariation, ...)

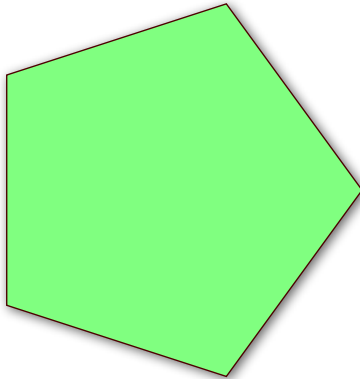
## ► Create

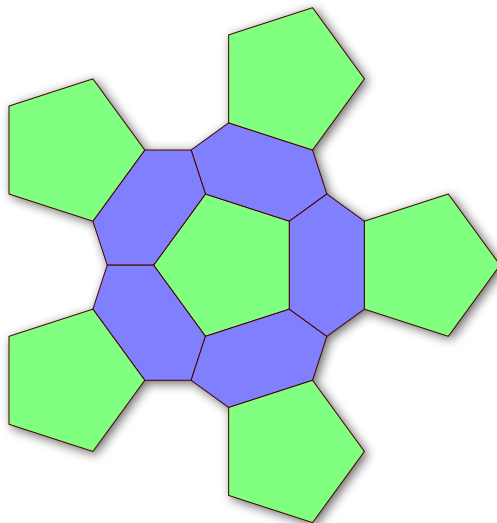
On all levels!

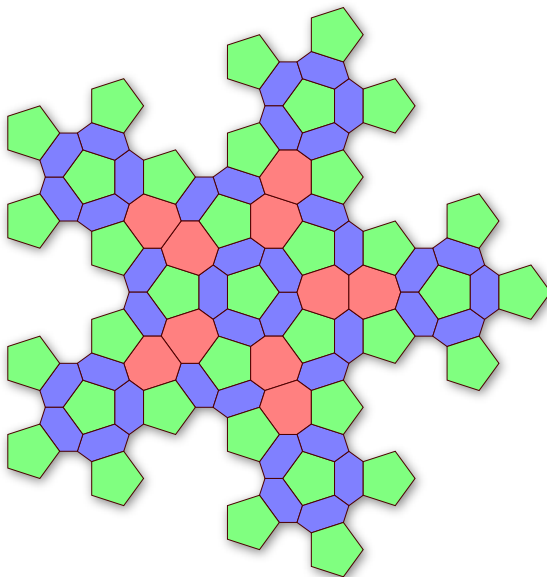
Patterns / Ornaments / Structures / ...

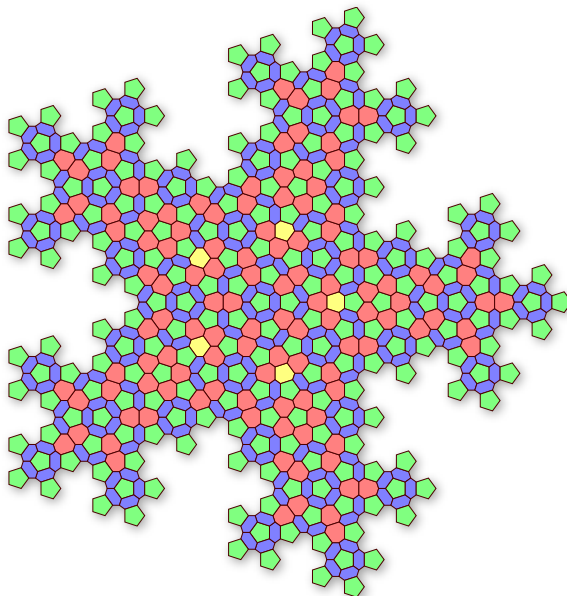
Collections of Interesting Things

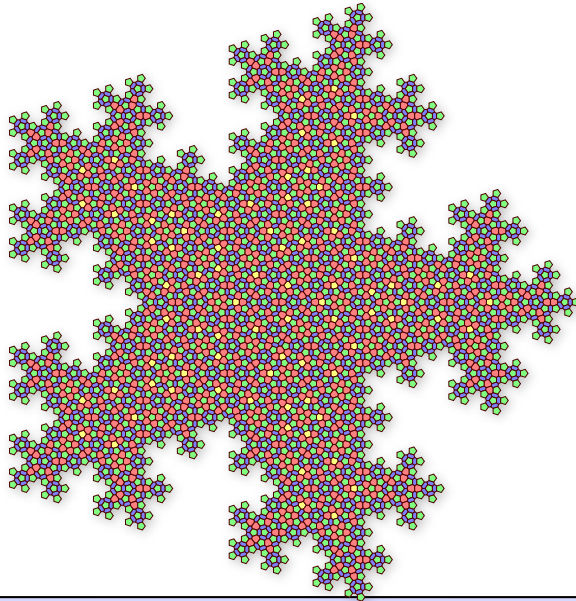


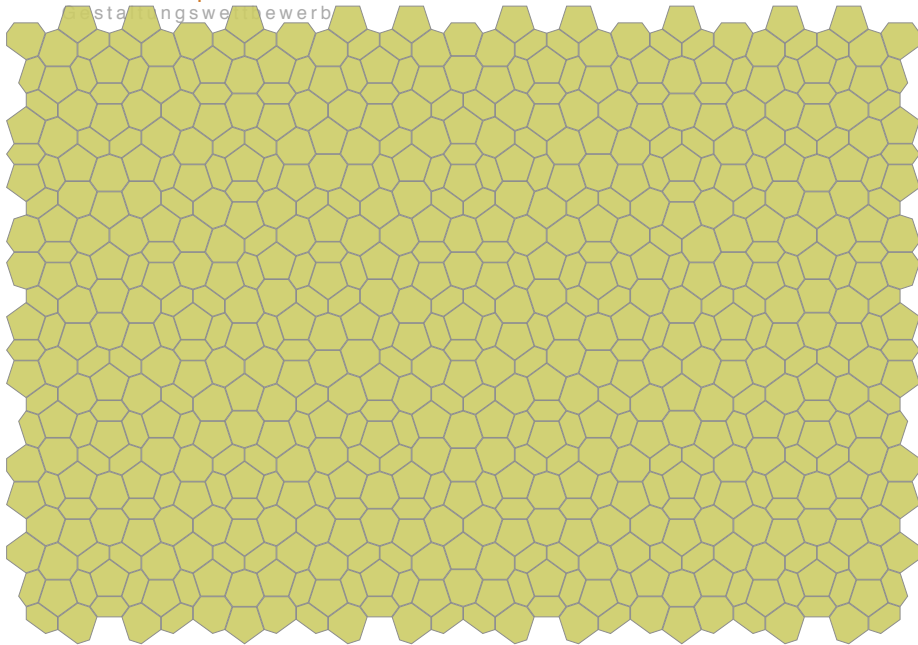














# Indra's Pearls

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► <http://cinderella.de/indra>

Part of



► <http://mathe-vital.de>

A wiki-based collection of high-quality OERs  
(Open Educational Resources) that  
complement lectures and books

# Wikis

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WIKIPEDIA  
*Die freie Enzyklopädie*

- ▶ Everybody knows Wikipedia
- ▶ All learning activities seem to end up in Wikipedia – good or bad?
- ▶ Web 2.0: Everybody can join – not really...
- ▶ New Paradigm: "Open"
- ▶ Necessary for us:  
**Editorial work** and **recognition of authors**

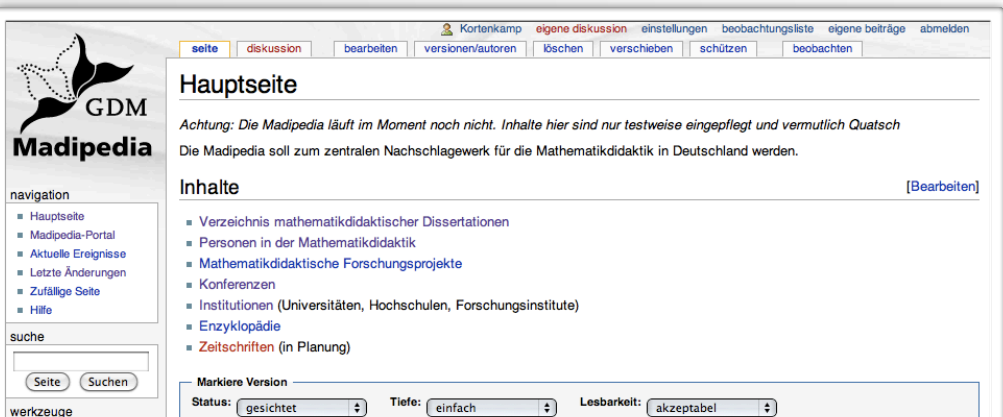
## German initiative in mathematics education accepting modern learning

Madipedia will become the central point of information  
about mathematics education research:

- ▶ Catalog of Ph.D. theses
- ▶ Persons in Mathematics Education
- ▶ Overview over Conferences and Projects
- ▶ Encyclopedia
- ▶ ...

Goal: Easy access to all information in math education for better research of everybody!

Currently in beta-testing and fine-tuning of concepts & software.

The screenshot shows the Madipedia.de website interface. At the top, there is a navigation bar with links: "Kortenkamp", "eigene diskussion", "einstellungen", "beobachtungsliste", "eigene beiträge", and "abmelden". Below this is a secondary navigation bar with buttons: "seite", "diskussion", "bearbeiten", "versionen/autoren", "löschen", "verschieben", "schützen", and "beobachten". The main content area is titled "Hauptseite" and contains a message: "Achtung: Die Madipedia läuft im Moment noch nicht. Inhalte hier sind nur testweise eingepflegt und vermutlich Quatsch. Die Madipedia soll zum zentralen Nachschlagewerk für die Mathematikdidaktik in Deutschland werden." Below this is a section titled "Inhalte" with a list of links: "Verzeichnis mathematikdidaktischer Dissertationen", "Personen in der Mathematikdidaktik", "Mathematikdidaktische Forschungsprojekte", "Konferenzen", "Institutionen (Universitäten, Hochschulen, Forschungsinstitute)", "Enzyklopädie", and "Zeitschriften (in Planung)". On the left side, there is a "navigation" section with links: "Hauptseite", "Madipedia-Portal", "Aktuelle Ereignisse", "Letzte Änderungen", "Zufällige Seite", and "Hilfe". Below this is a "suche" section with a search bar and buttons "Seite" and "Suchen". At the bottom, there is a "werkzeuge" section with a "Markiere Version" section and a "Lesbarkeit" section with a dropdown menu set to "akzeptabel".

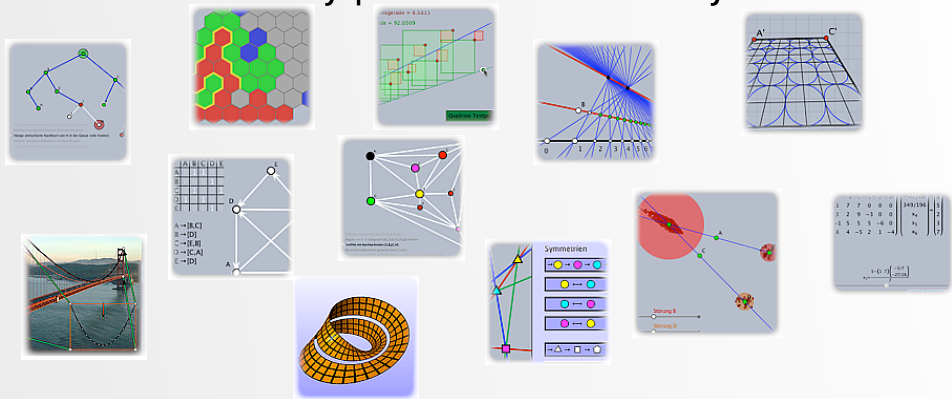
# Klein-Wiki (Madipedia.org?)



- ▶ Should be orthogonal to the book's topics
- ▶ High-Quality and reviewed content
- ▶ Citation-safe source
- ▶ Authors are visible and recognized
- ▶ Interactive content
- ▶ Editorial board needed
- ▶ Mathe-Vital.de as a basis?

# How to create content

- Formalisation through programming
- Banchoff: "some students wrote some code"
- There are very powerful and easy tools



# Built into Cinderella: CindyScript

---

## Functional Programming Language:

### ► non-typed, list-supporting

```
numbers = 1..100; i = 0;  
odd numbers = apply(numbers, 2*#+1);
```

### ► definitions

```
divisors(x) := select(1..x, mod(x, #)==0);  
prime(p) := (length(divisors(p))==2);
```

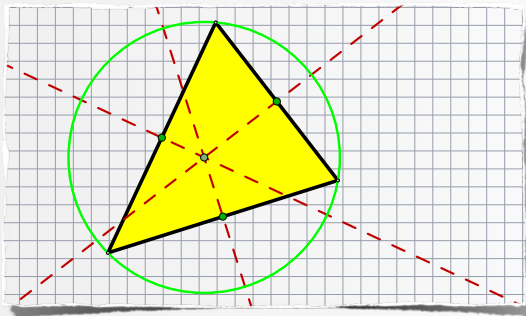
### ► loops, drawing, ...

```
odd primes = select(odd numbers, prime(#));  
forall(odd primes, draw([i, #])); i=i+1;
```

# Teaching?

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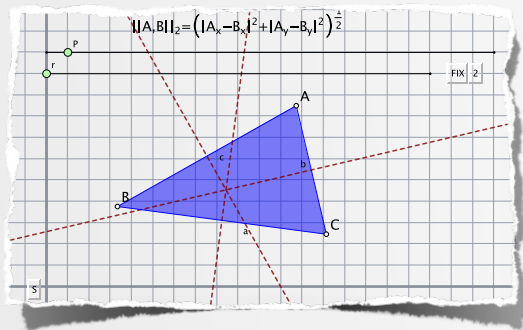
- Is it really necessary to have a programming language inside a teaching tool?





# Teaching?

- Is it really necessary to have a programming language inside a teaching tool?



Yes: Because  
there is more  
than points, lines  
and circles in life!



<http://twitter.com/ukor>



Cheap GPS. (from: xkcd.com)

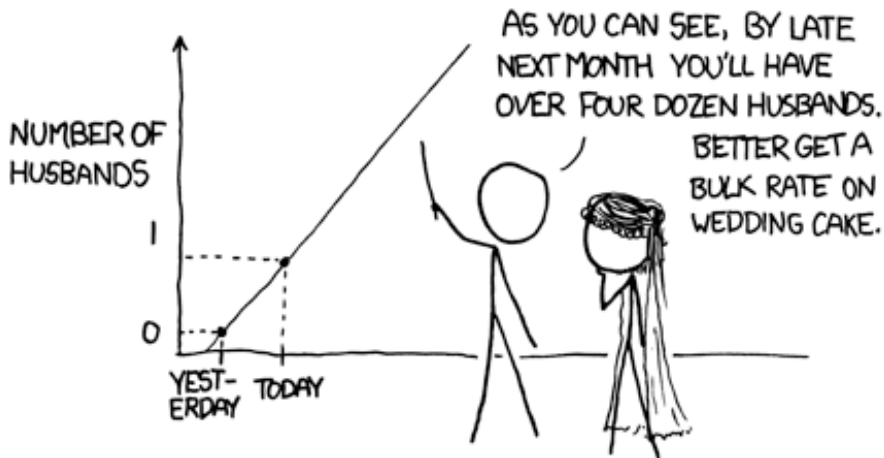
<http://mathe-vital.de>

<http://cinderella.de>

<http://kortenkamp.net>

<http://madipedia.de>

## MY HOBBY: EXTRAPOLATING






# Moore's Law

---

(Wrong, but more spectacular version)

Computing power doubles every 18 months.

Or: Every two years everything becomes  
**twice as good**

	Harddisk	iPod	MHz	Memory	E-Mail
1980			4	16.384	
1981					
1982					
1983					
1984					
1985					
1986					
1987					
1988					
1989					
1990	60.000.000		25	1.048.576	
1991					
1992					
1993					
1994					
1995					432
1996					465
1997					400
1998					868
1999					1.472
2000	60.000.000.000		450	536.870.912	1.364
2001					2.007
2002	120.000.000.000	5.000.000.000			4.667
2003			800	1.073.741.824	2.762
2004					4.250
2005			1.670		8.621
2006					13.271
2007	320.000.000.000	16.000.000.000	3.660		13.996
2008				3.221.225.472	14.519
2009	1.000.000.000.000	32.000.000.000	4.800		15.135

# On a log scale it's correct

