The Impact of Computer Science on Mathematics Education

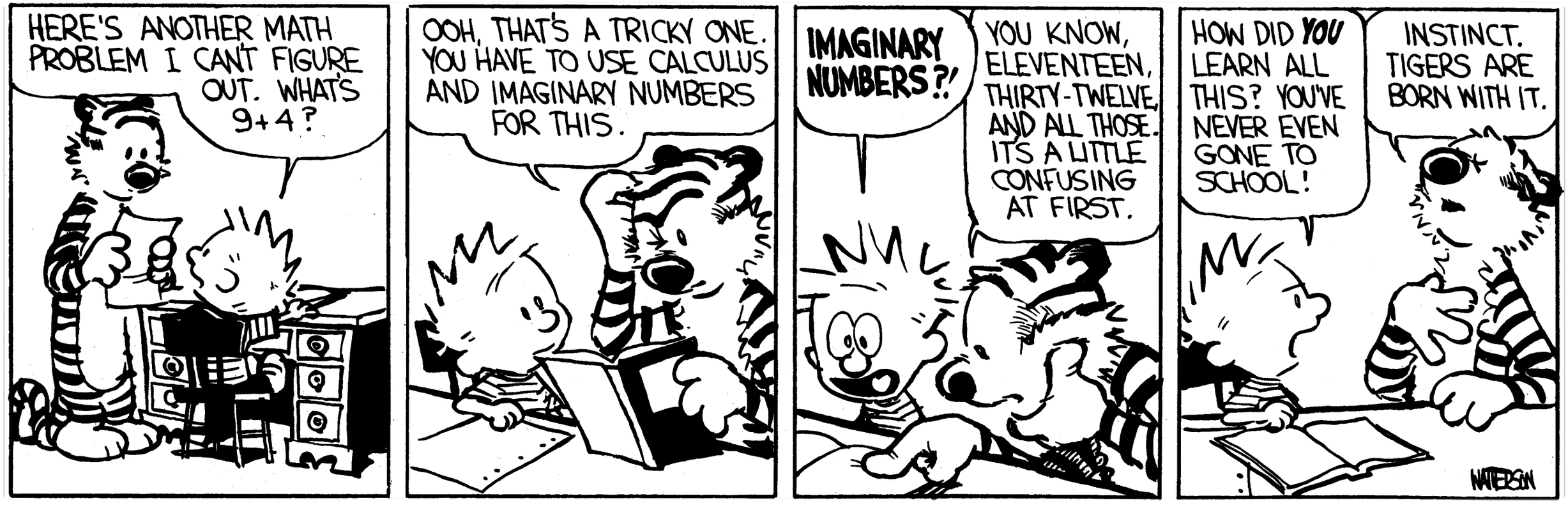
Ulrich Kortenkamp
CERMAT, University of Education Karlsruhe
DM@MD, Funchal, Madeira, Oct. 2, 2009

Cinderella.2
Math in Motion

I2G
INTERGEO
Interoperable
Interactive
Geometry for Europe

CERMAT - Centre for
Educational Research in
Mathematics and Technology

The Impact of Computer Science on Mathematics Education



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Zwei Probleme der inhomogenen diophantischen Approximation

Jürgen Bokowski
1973

Obere Schranke zur Gitterpunktsanzahl konvexer Körper

Jürgen Richter-Gebert
1992

On the Realizability Problem of Combinatorial geometries-Decision Methods

Ulrich Kortenkamp
1999

Foundations of Dynamic Geometry

er Charakteristiken

970 1980 1990 2000 2010 2020 Jahr der Dissertation

Dynamic Geometry

Objects

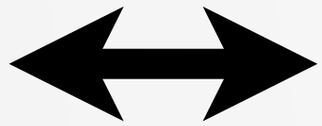
Points

Lines

Circles, Conics

Numbers

...



Relations

incidence

orthogonality

parallelism

is x-coordinate

...

Interactive Geometry

Objects

Points
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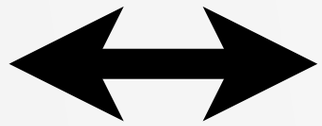

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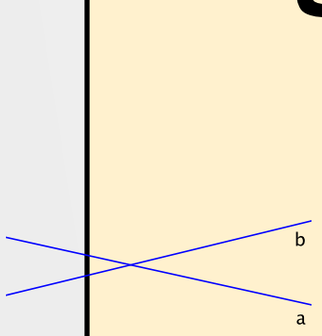

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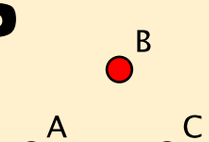

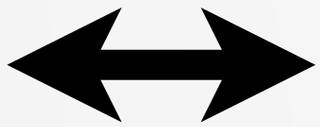

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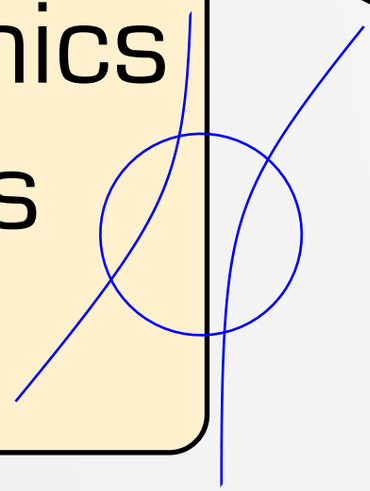
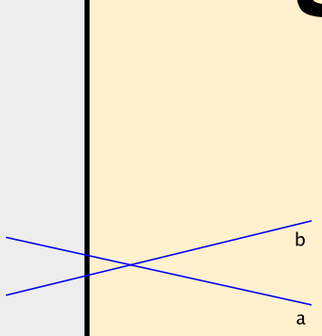
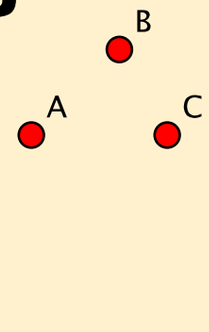


Interactive Geometry

Objects



Points



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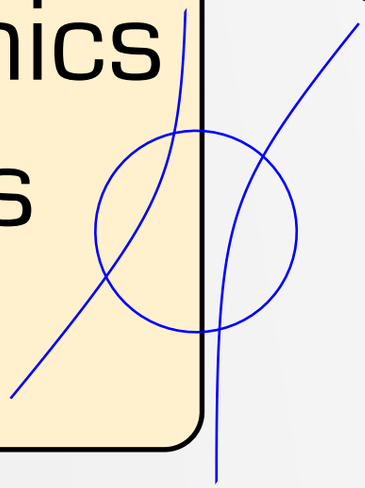

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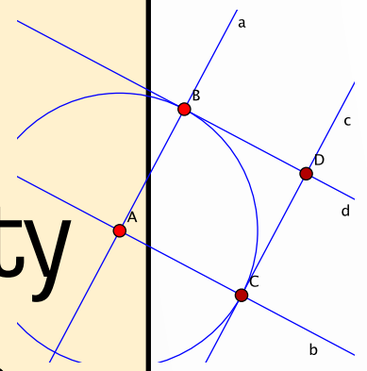


Easy, isn‘t it?

Easy, isn‘t it?

No.

Demo

Easy, isn‘t it?

No.

Easy, isn‘t it?

It‘s even worse.

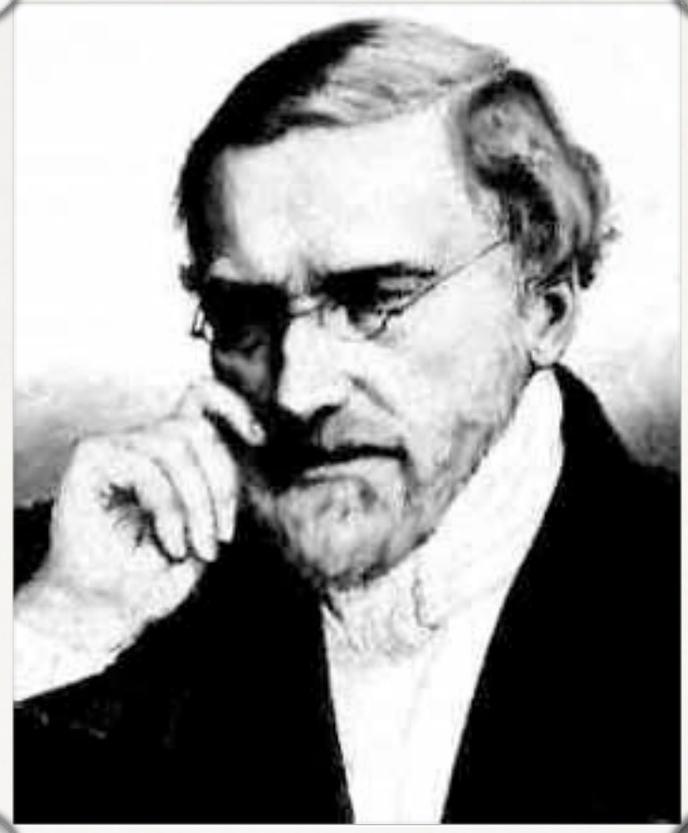
Demo

Solution:
Not easy, but nice.

What we want

Continuity Principle by Poncelet

- “A relation known to hold with sufficient generality for a given figure also holds for all other figures that may be derived from it by continuous variation”

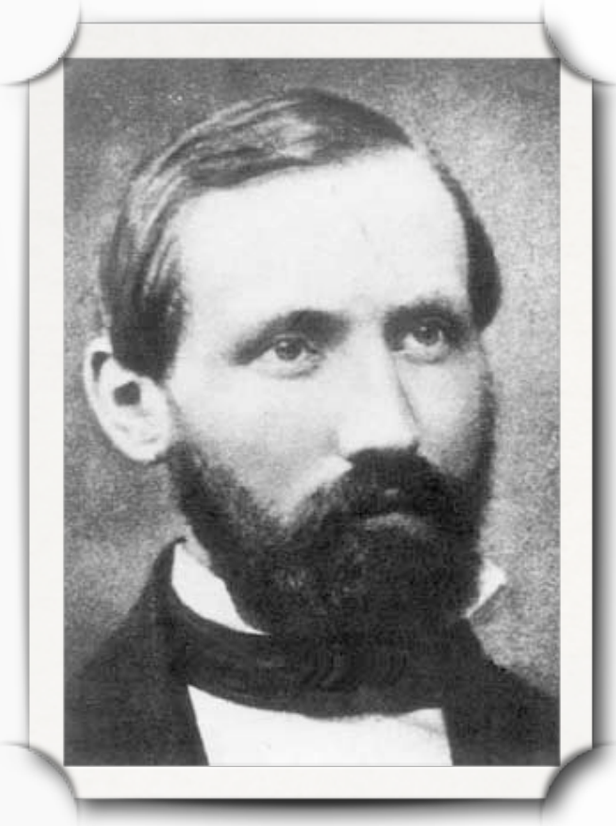


Victor Poncelet

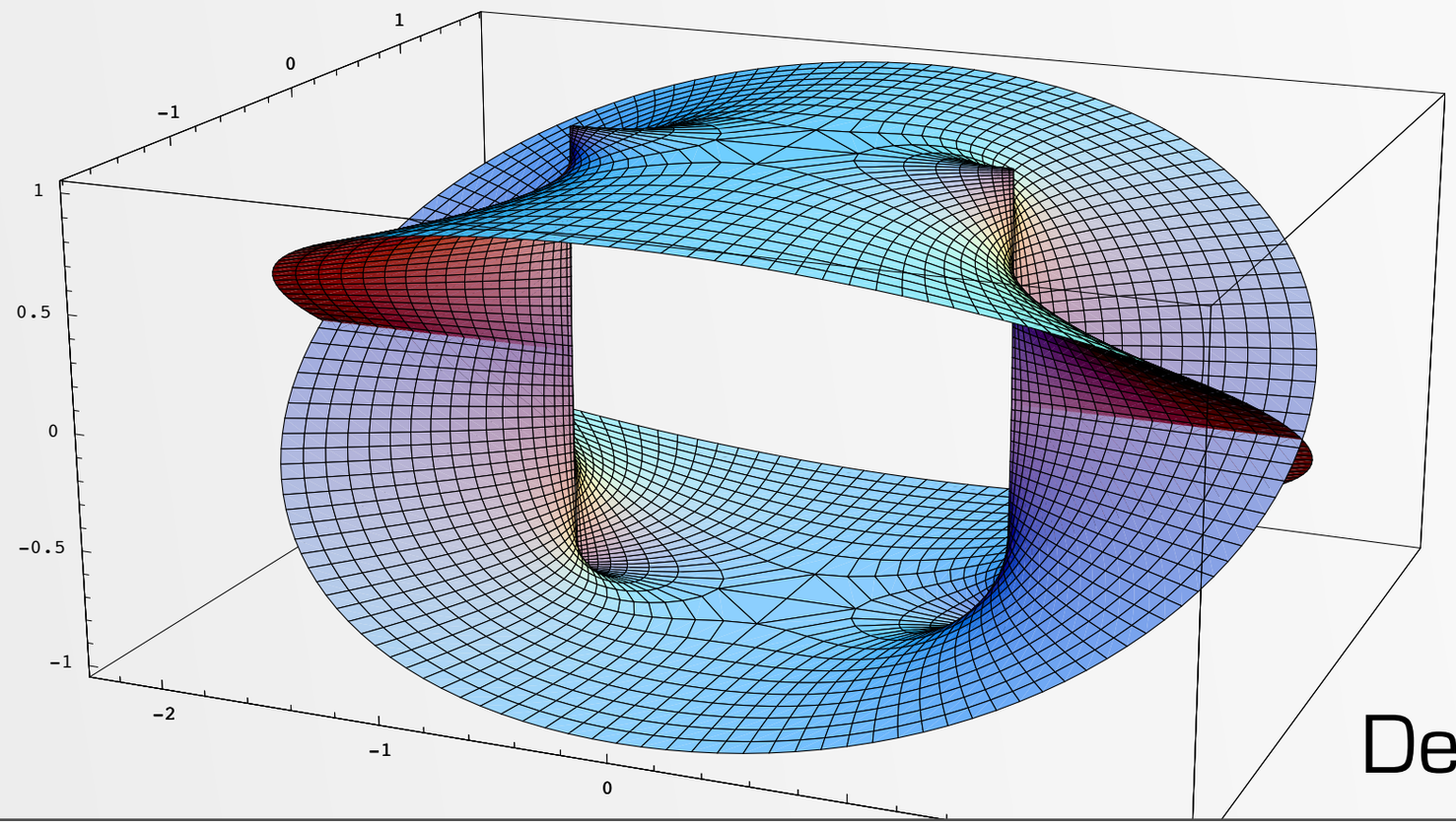
Small changes in input lead to small changes in output

What we have to do

- Work with complex homogeneous coordinates
- Do analytic continuations on a Riemann surface



Bernhard Riemann



Details: Broser 2008

Underlying algorithm

- Use “close-to” tracing algorithm

- Avoid singularities by complex detours

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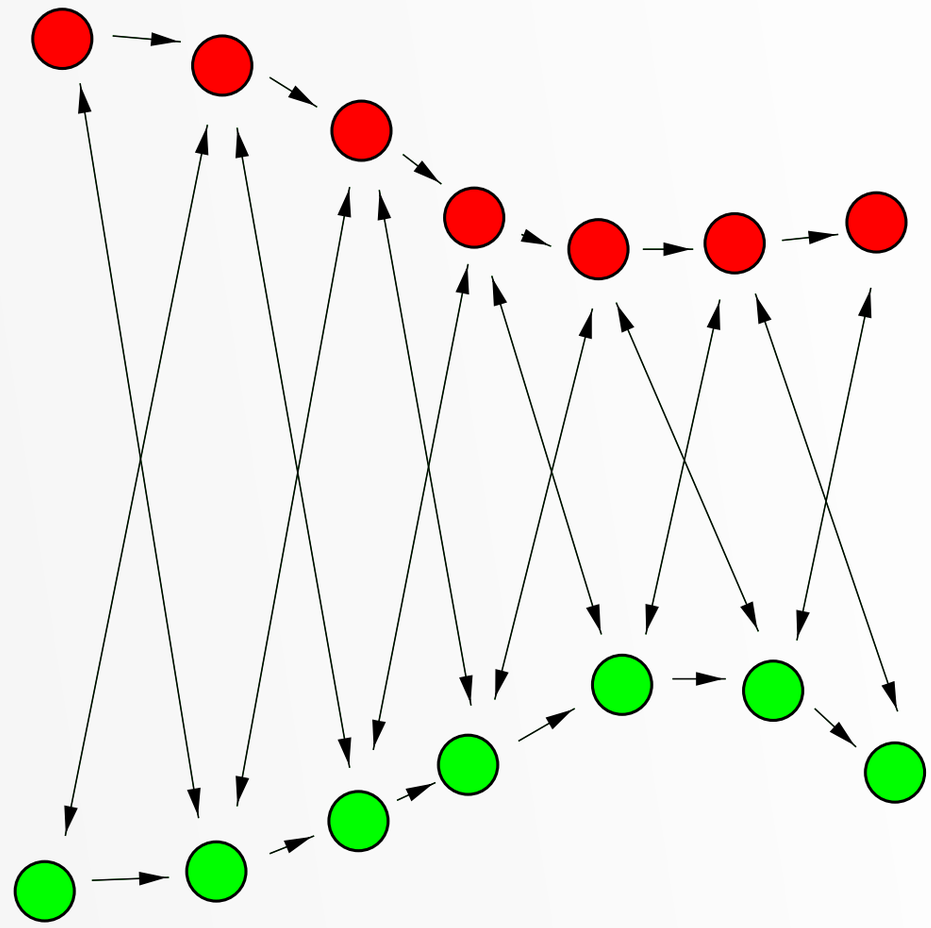

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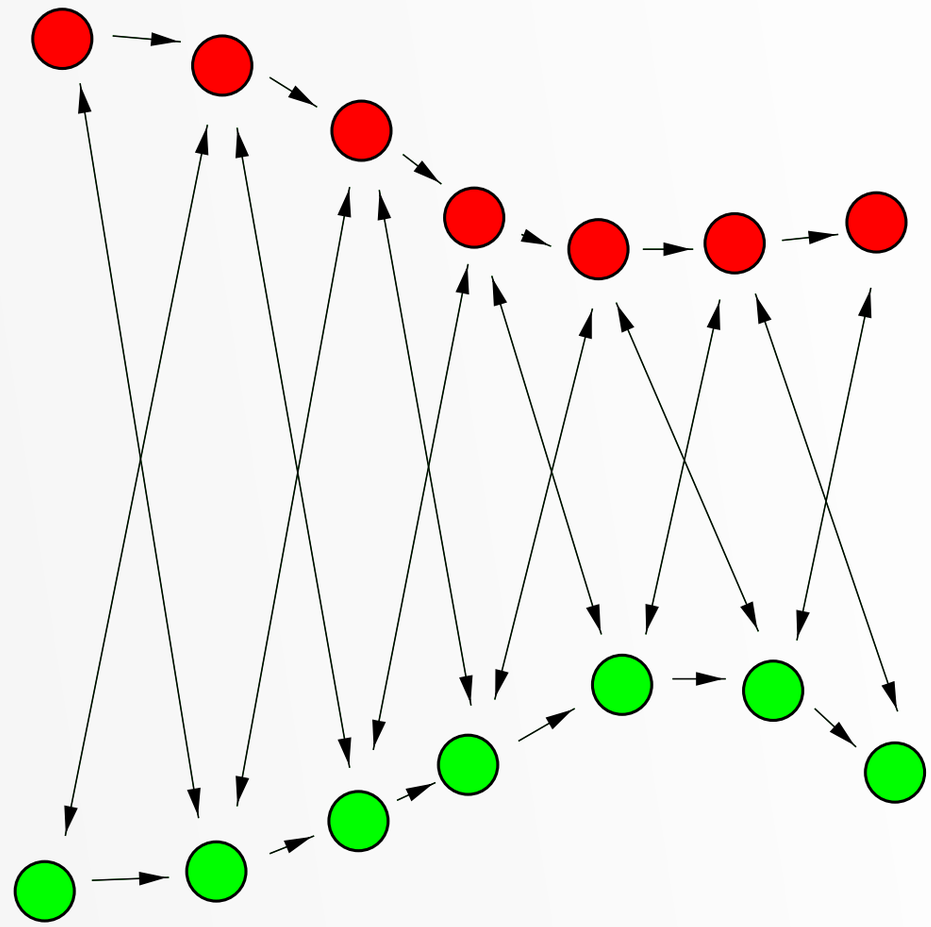

see Video on Vimeo

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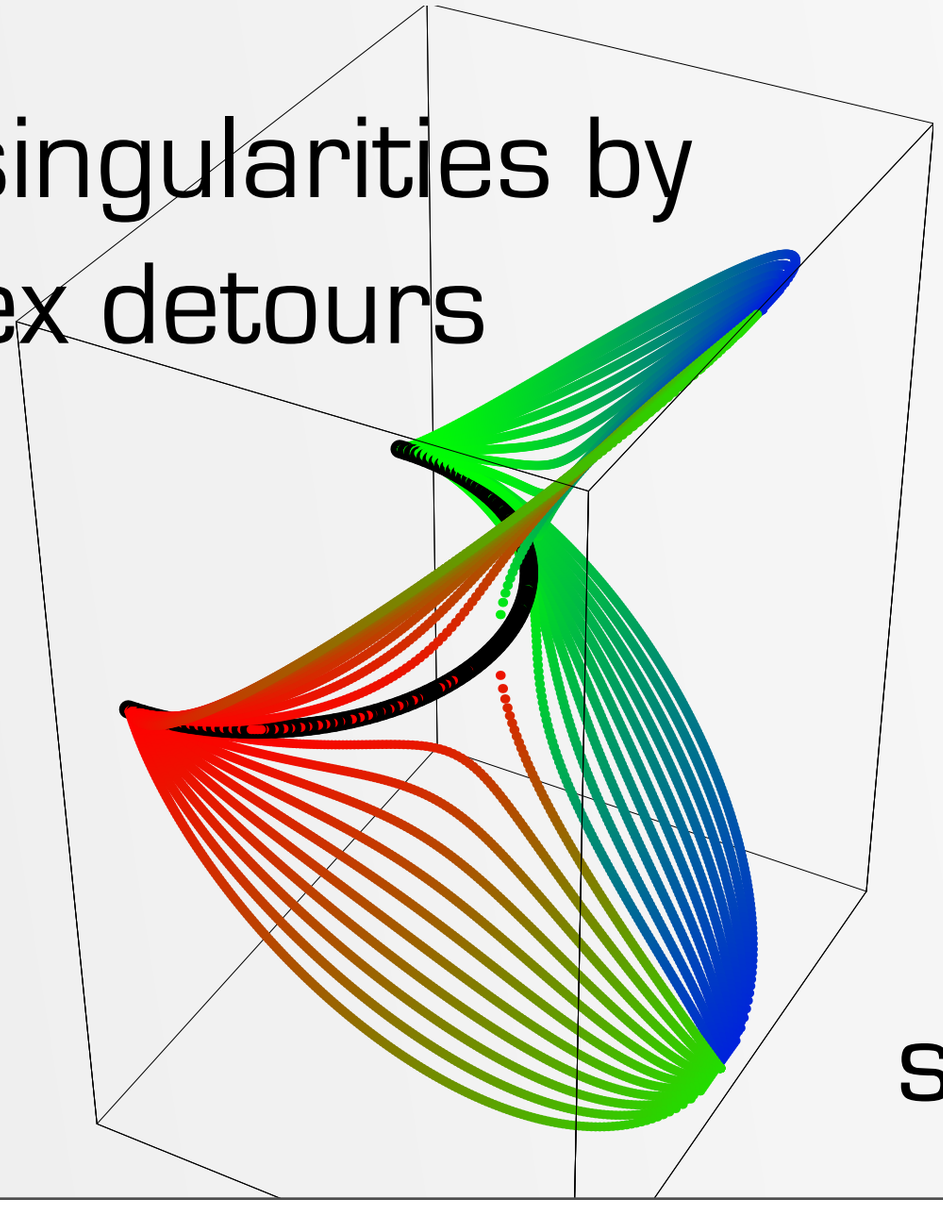

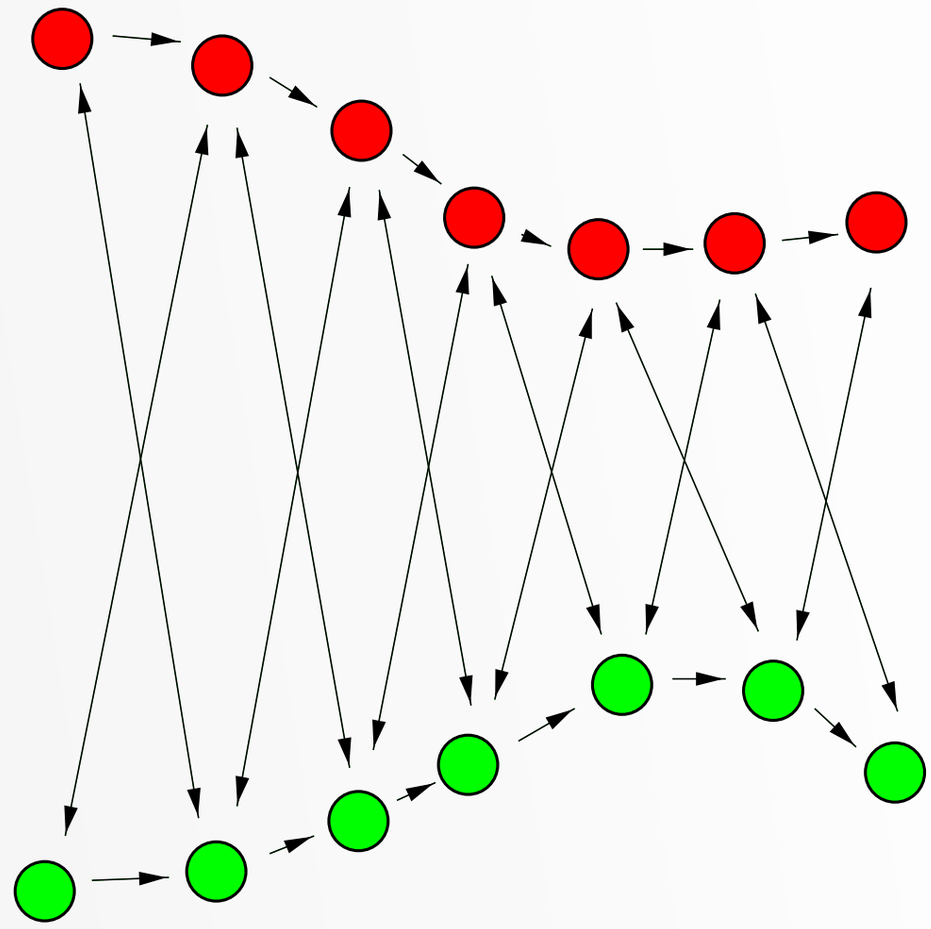

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Consequences

- All dependent elements behave continuously (in fact analytically)
- We have a notion of geometric theorem
- We can check theorems numerically
- Everything works “as expected” (for mathematicians, which is not what most people expect)

Felix Klein

turali – De Absoluta Dei Simplicitate,
id est Rempublicam In Microcosmo Conspicuam

ann Christoph Wichmannshausen
35

Disputationem Moralem De Divortiis Secundum Jus Naturae

Christian Hausen
1713

De corpore scissuris figurisque non cruetando ductu

Abraham Gotthelf Kästner
1739

Theoria radicum in aequationibus

Johann Friedrich Pfaff
1786

Commentatio de ortibus et occasibus s
classicos commemoratis

Carl Friedrich Gauß
1799

Demonstratio nova th
algebraicam rationale
reales primi vel secun

Christian Ludwig G
1812

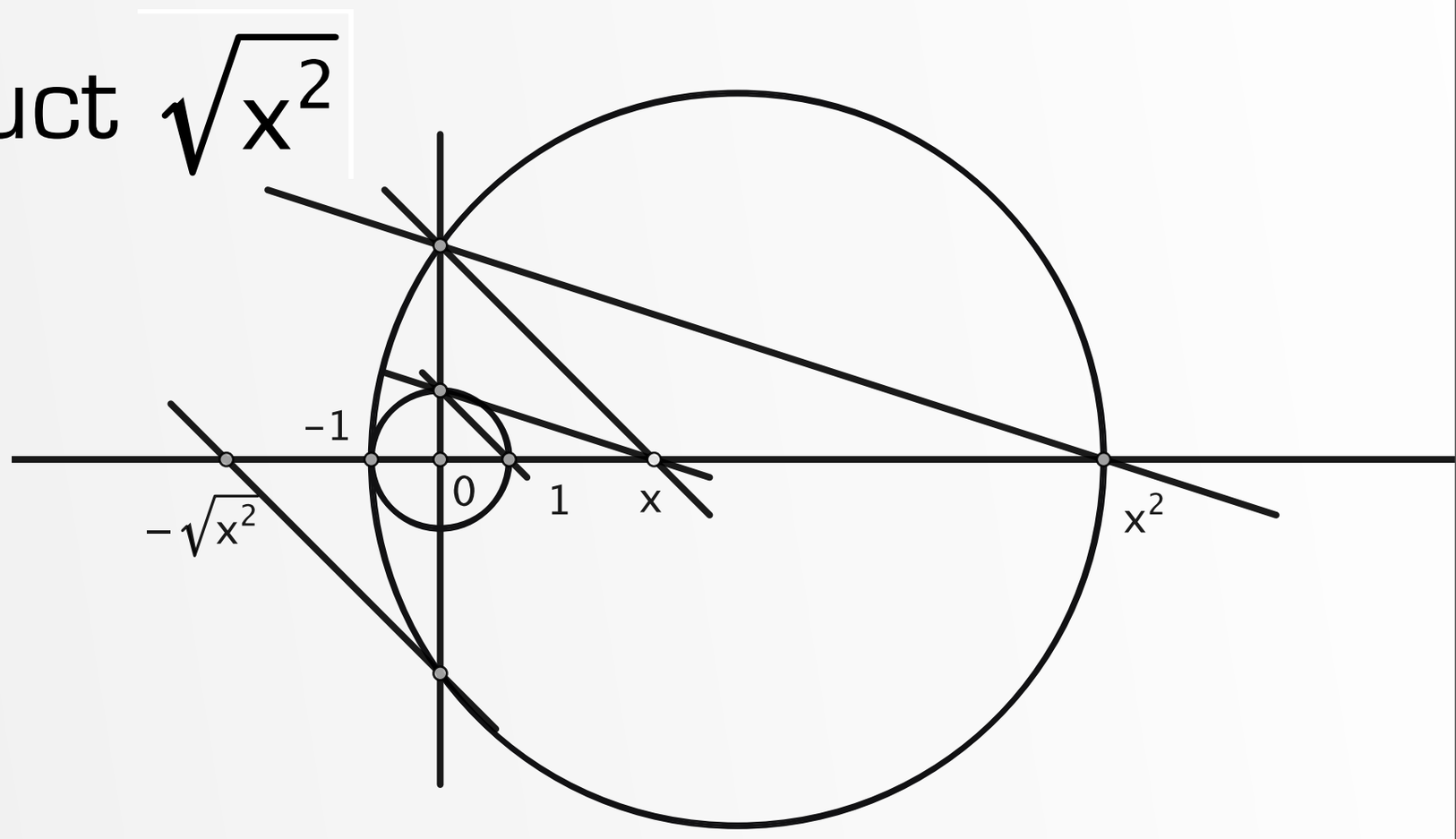
Method
parallac
die

Julius
1823

690 1700 1710 1720 1730 1740 1750 1760 1770 1780 1790 1800 1810 1820 1

Functions and Roots

- von-Staudt-Constructions: add & multiply (field structure in Projective Geometry)
- Euclid: find square root
- we can construct $\sqrt{x^2}$

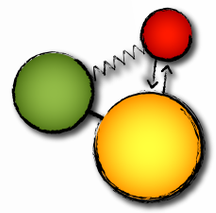


Similar: Oriented Areas

- Talk of Thomas Banchoff
- Absolute value function is not analytic
- If we need a notion of "Theorem" we better avoid it
- If we allowed it, we could construct

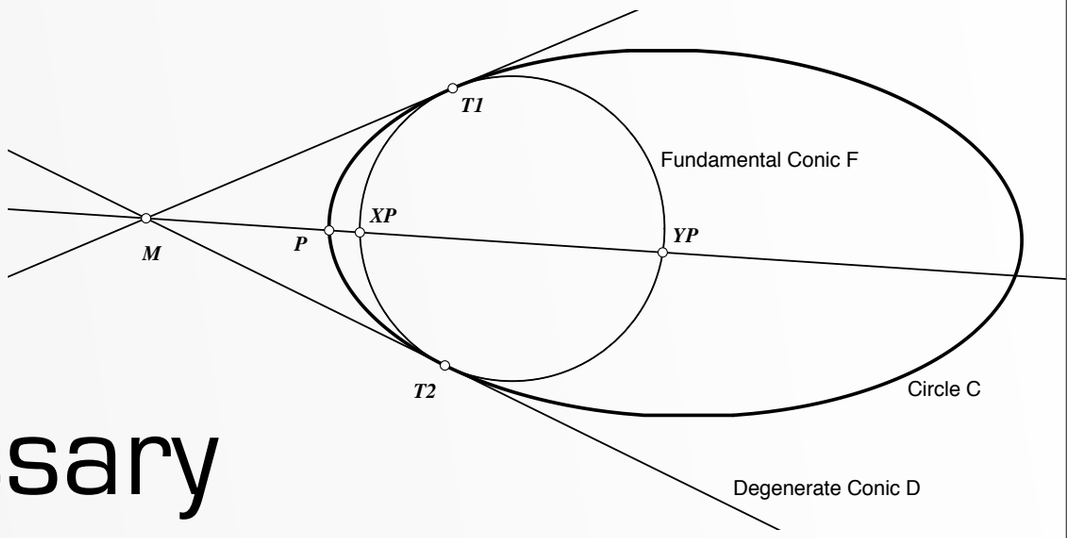
$$\frac{|x|}{x} = \begin{cases} 1 & \text{for} \quad x>0 \\ -1 & \text{for} \quad x<0 \end{cases}$$

Similar: Angles & Distances



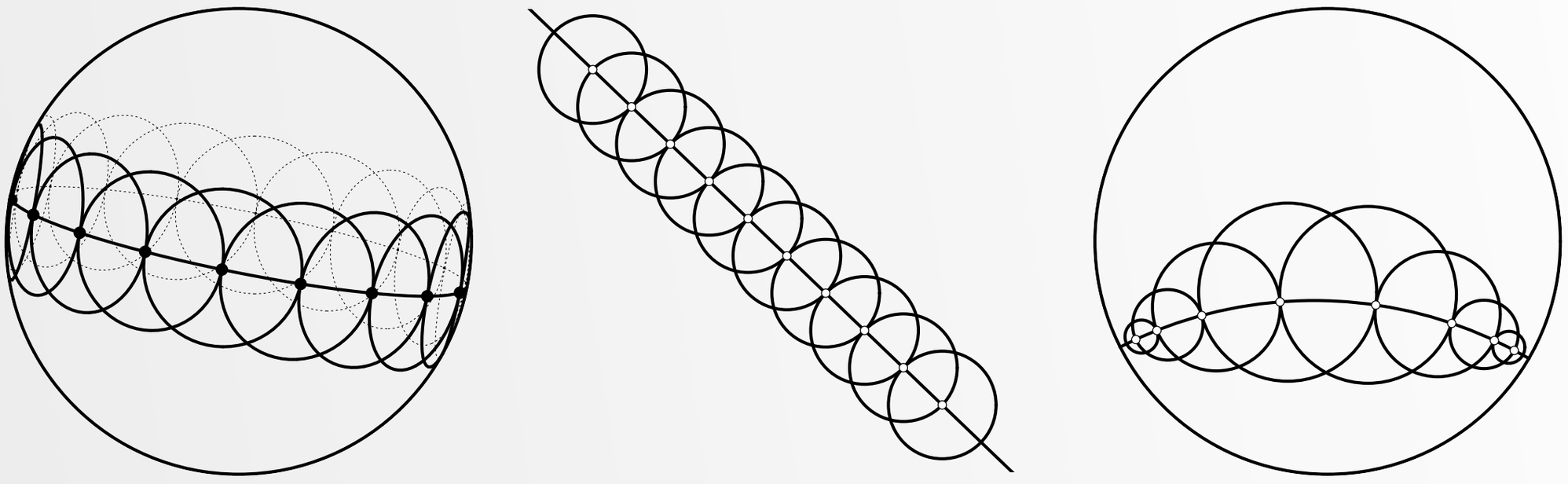
- Unified way of measuring:

  Cayley-Klein Geometries



- Real understanding necessary

- "Normal way" is a special case



Daß es mehrere Winkelbegriffe gibt, ist schon früher zur Sprache gekommen. Manche Didaktiker wollen uns davon überzeugen, daß nur einer der richtige sei. Ordnungsliebe ist lobenswert, aber sie sollte nicht so weit gehen, daß man wichtige Begriffe verbietet, weil sie nicht ins System passen.

Mathematik als pädagogische Aufgabe,
Hans Freudenthal, Klett 1973

It was mentioned earlier that there are several notions of angle. Some educationists want to convince us that only one of them is the correct one.
Tidiness is wonderful, but it should not be exaggerated if this leads to banning important notions just because they don't fit into the system.

Mathematik als pädagogische Aufgabe,
Hans Freudenthal, Klett 1973

Concepts & Processes

- **Change**

  In every meaning of the word!

  Wonderful Example:

  Hyperbolization of Ornaments

  (von Gagern & Richter-Gebert)

- **http://www.combinatorics.org/Volume_16/PDF/v16i2r12.pdf**

  needs Hypergeometric Functions (Klein!)

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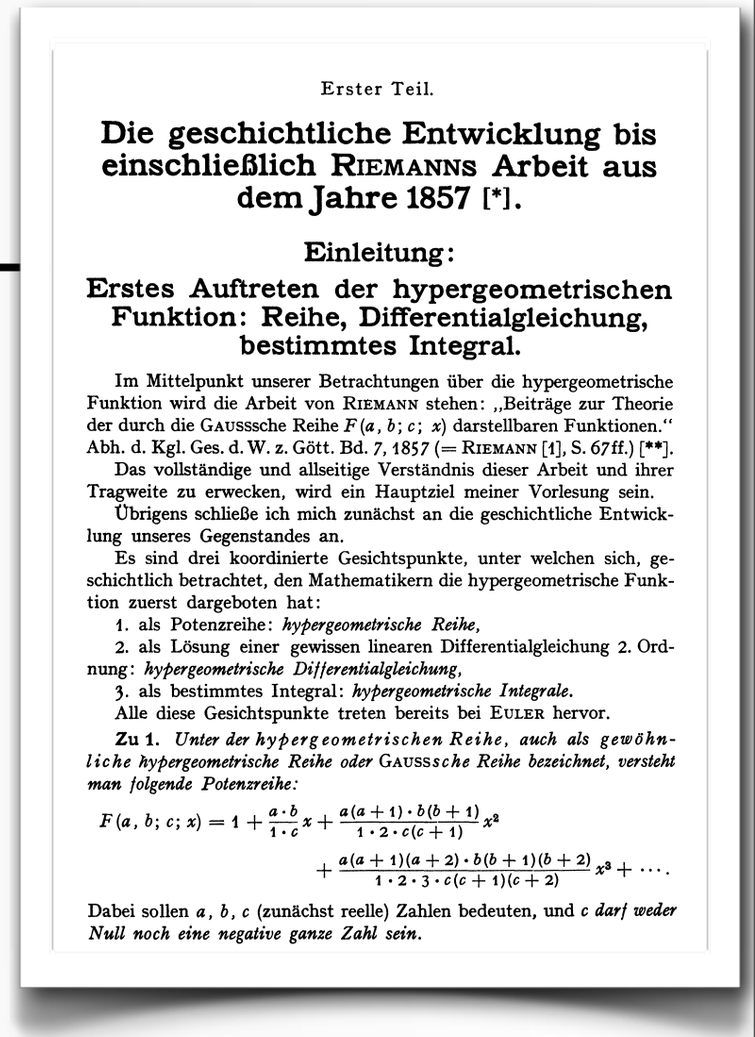


Erster Teil.

Die geschichtliche Entwicklung bis einschließlich RIEMANNS Arbeit aus dem Jahre 1857 [*].

Einleitung:

Erstes Auftreten der hypergeometrischen Funktion: Reihe, Differentialgleichung, bestimmtes Integral.

Im Mittelpunkt unserer Betrachtungen über die hypergeometrische Funktion wird die Arbeit von RIEMANN stehen: „Beiträge zur Theorie der durch die Gausssche Reihe $F(a, b; c; x)$ darstellbaren Funktionen." Abh. d. Kgl. Ges. d. W. z. Gött. Bd. 7, 1857 (= RIEMANN [1], S. 67ff.) [**].

Das vollständige und allseitige Verständnis dieser Arbeit und ihrer Tragweite zu erwecken, wird ein Hauptziel meiner Vorlesung sein.

Übrigens schließe ich mich zunächst an die geschichtliche Entwicklung unseres Gegenstandes an.

Es sind drei koordinierte Gesichtspunkte, unter welchen sich, geschichtlich betrachtet, den Mathematikern die hypergeometrische Funktion zuerst dargeboten hat:

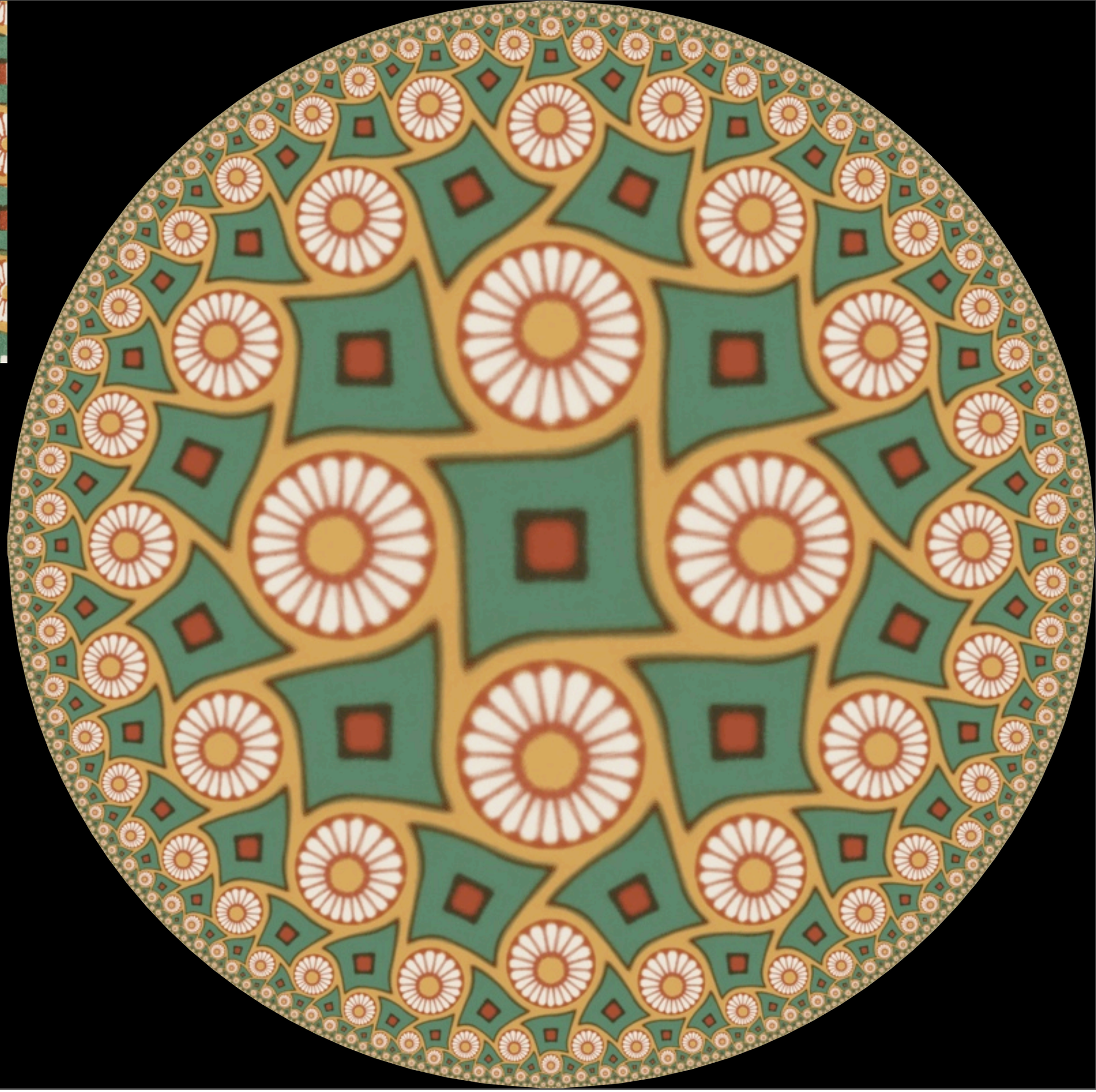
1. als Potenzreihe: *hypergeometrische Reihe*,
2. als Lösung einer gewissen linearen Differentialgleichung 2. Ordnung: *hypergeometrische Differentialgleichung*,
3. als bestimmtes Integral: *hypergeometrische Integrale*.

Alle diese Gesichtspunkte treten bereits bei EULER hervor.

Zu 1. *Unter der hypergeometrischen Reihe, auch als gewöhnliche hypergeometrische Reihe oder* GAUSSsche *Reihe bezeichnet, versteht man folgende Potenzreihe:*

$$F(a, b; c; x) = 1 + \frac{a \cdot b}{1 \cdot c} x + \frac{a(a+1) \cdot b(b+1)}{1 \cdot 2 \cdot c(c+1)} x^2 + \frac{a(a+1)(a+2) \cdot b(b+1)(b+2)}{1 \cdot 2 \cdot 3 \cdot c(c+1)(c+2)} x^3 + \cdots.$$

Dabei sollen a, b, c (zunächst reelle) Zahlen bedeuten, und c *darf weder Null noch eine negative ganze Zahl sein.*



by von Gagern &
Richter-Gebert

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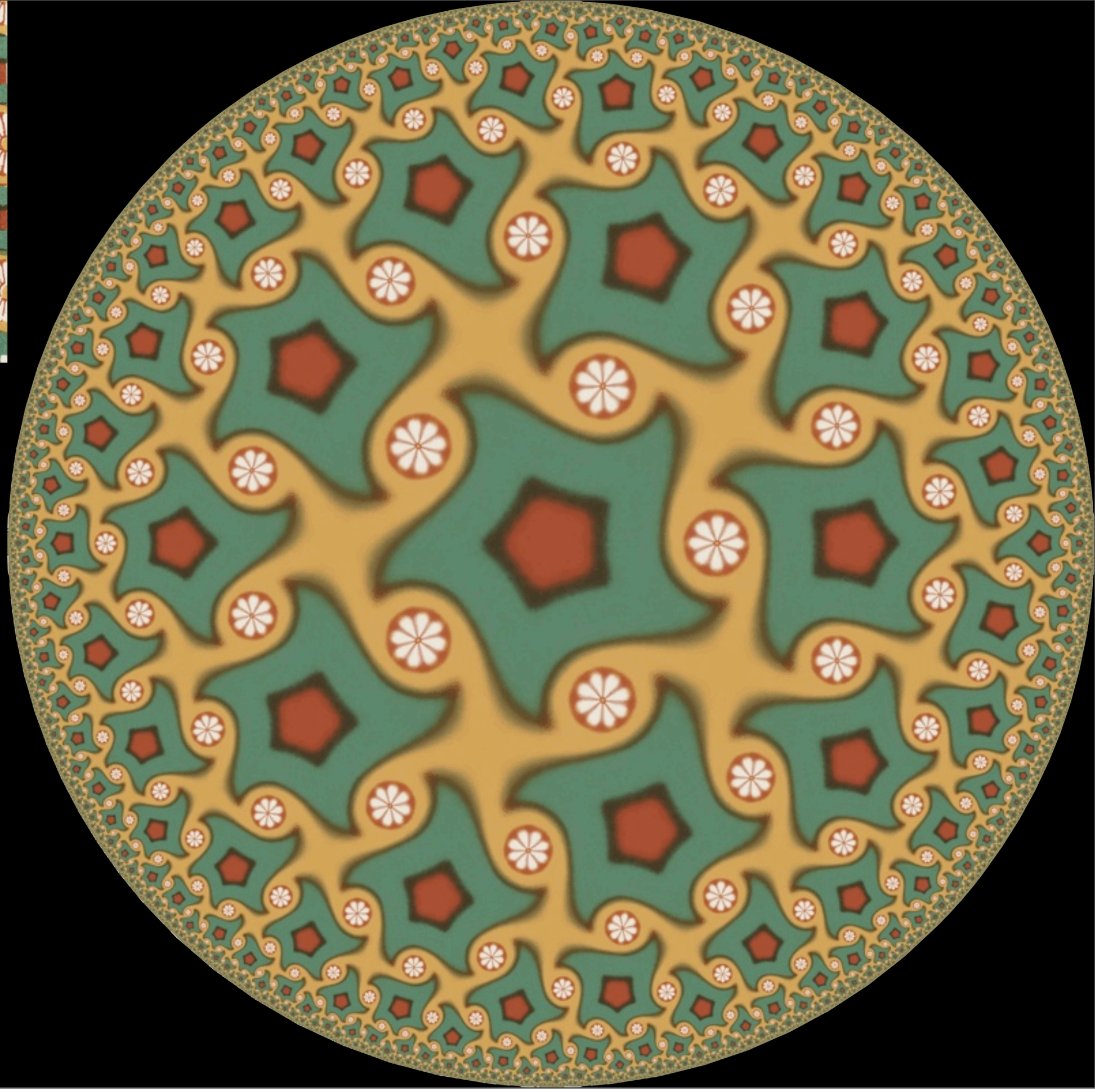

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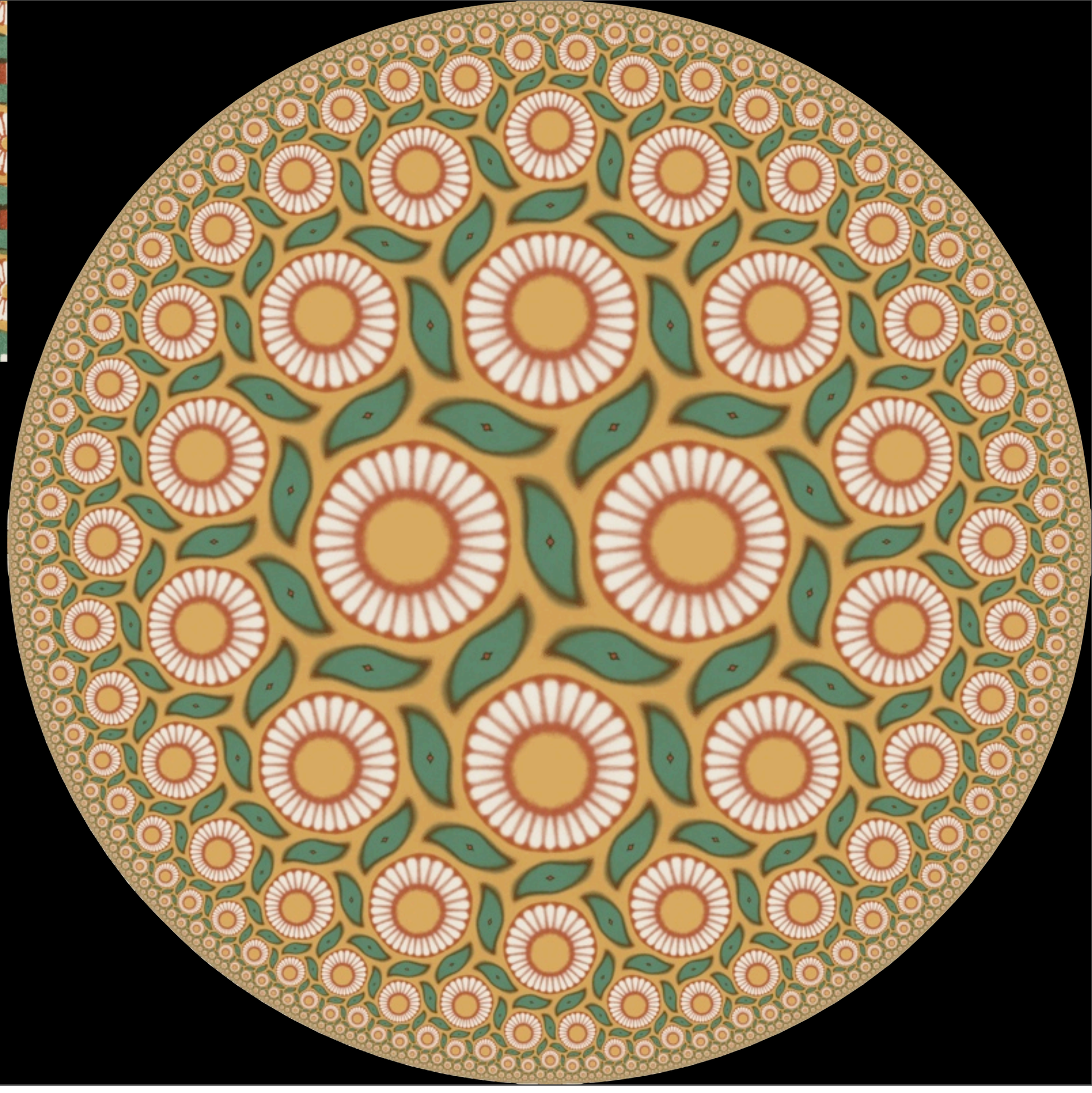

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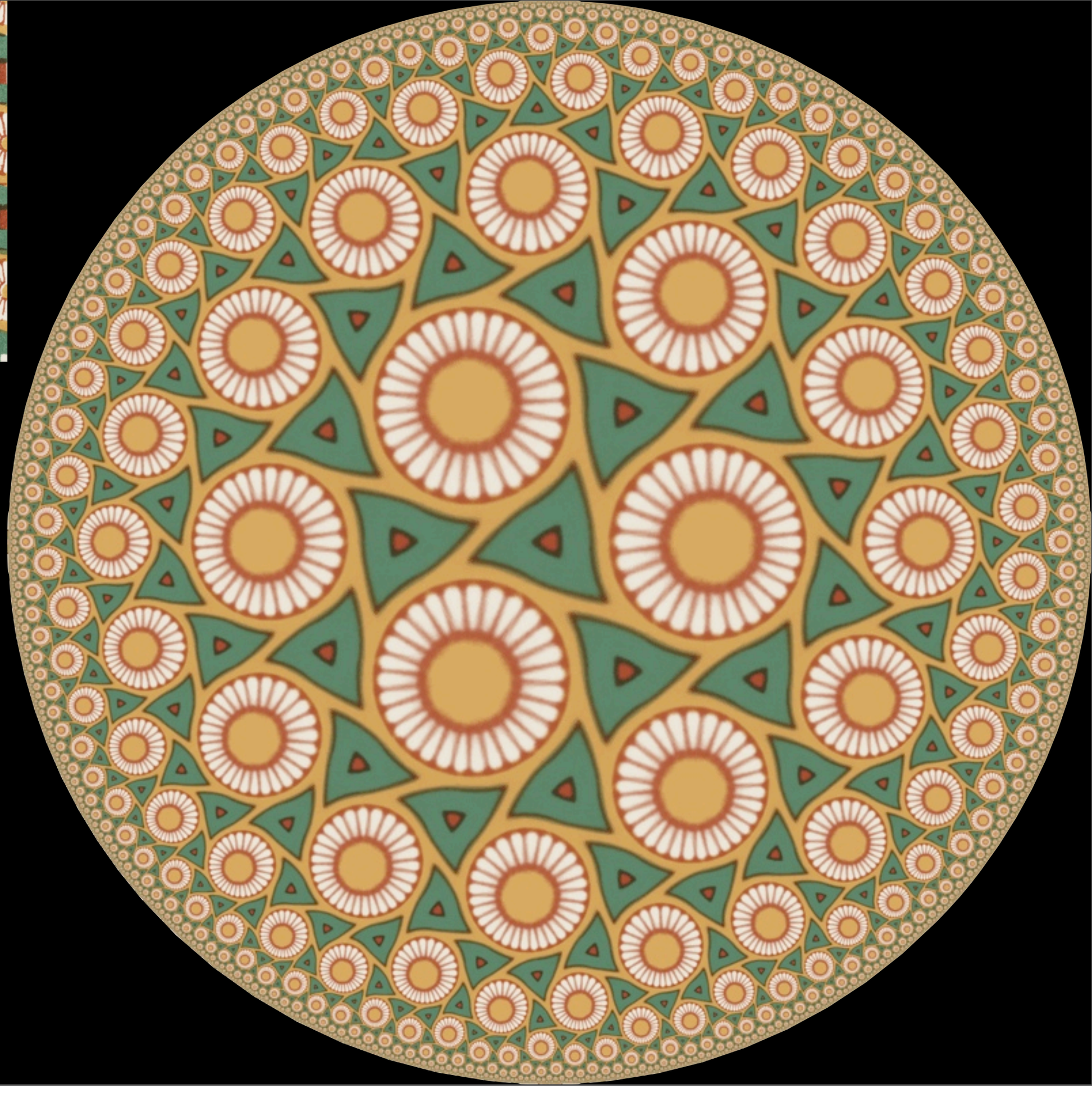

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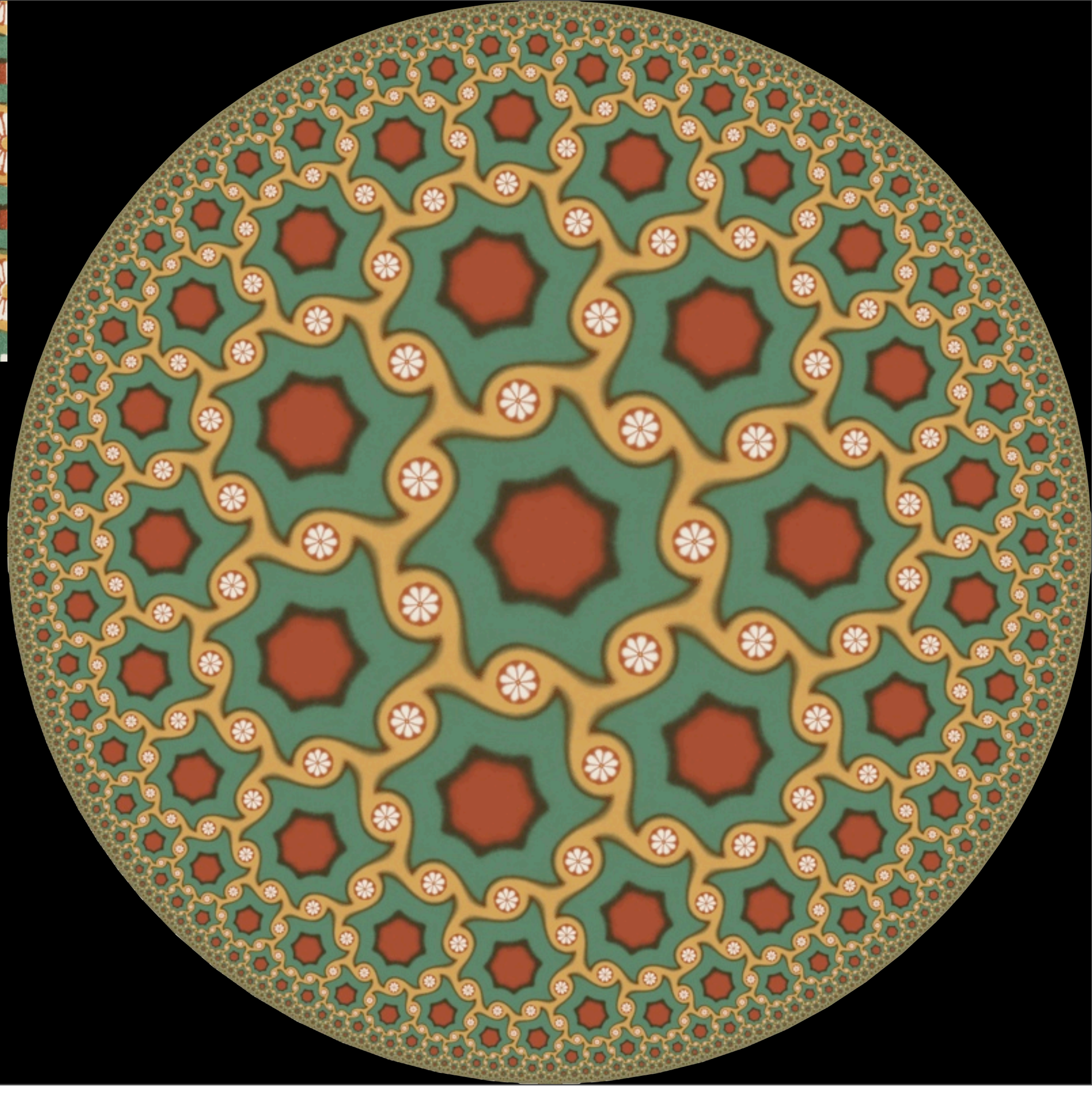

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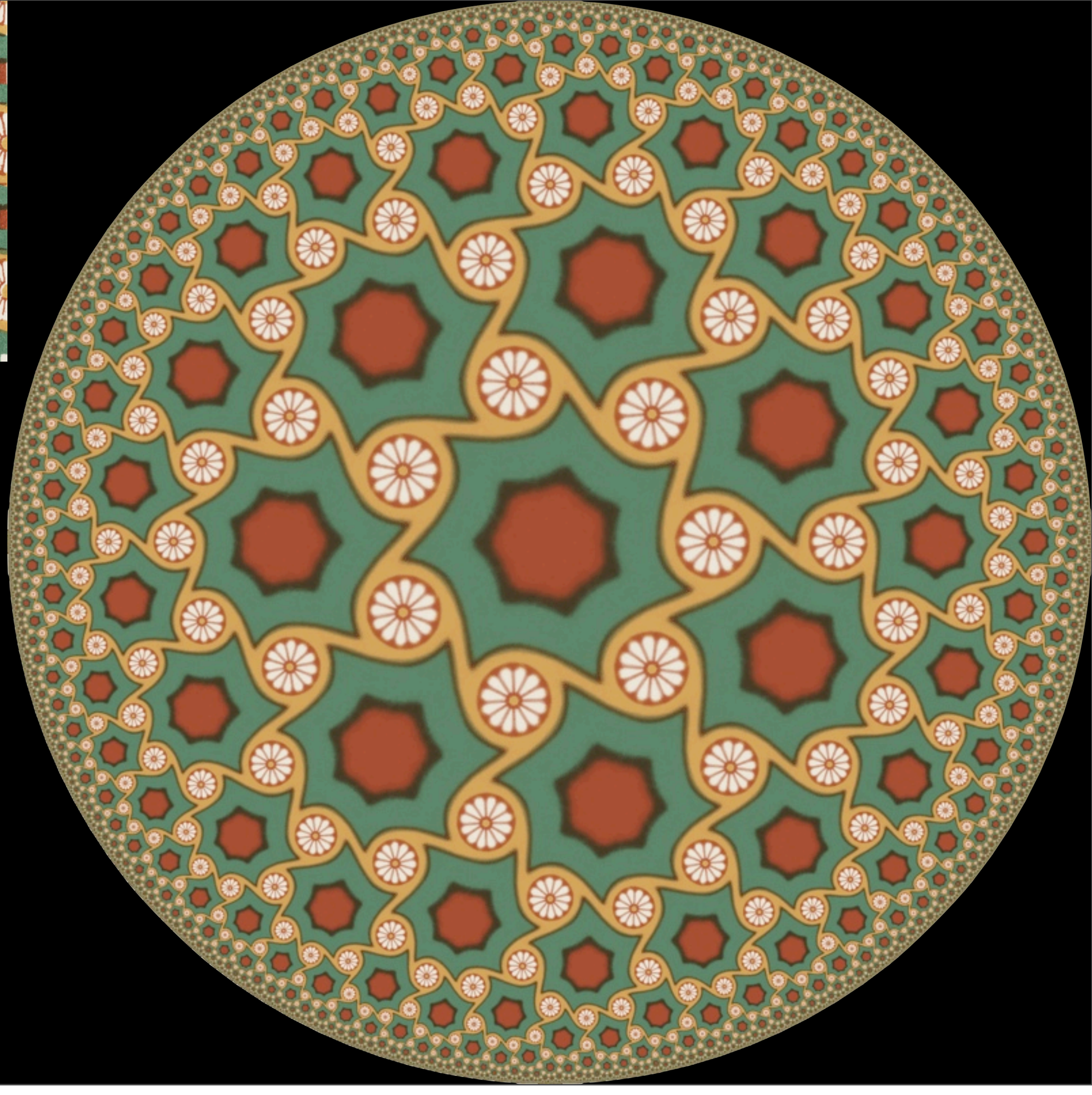

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Concepts & Processes

- **Change**

  In every meaning of the word!

  On every level (variation, metavariation, ...)

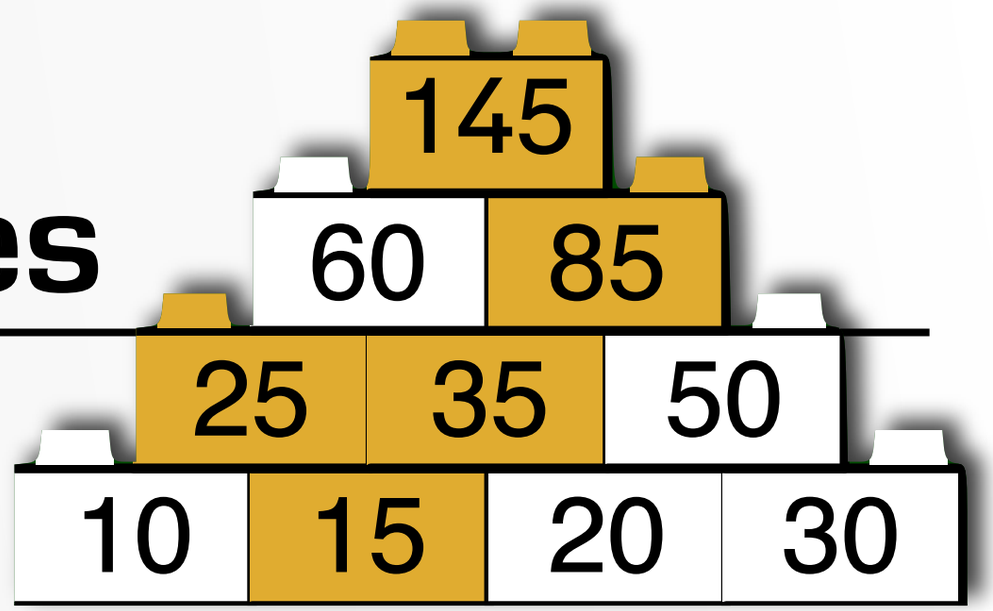
- **Create**

  On all levels!

  Patterns / Ornaments / Structures / ...

  Collections of Interesting Things

Concepts & Processes



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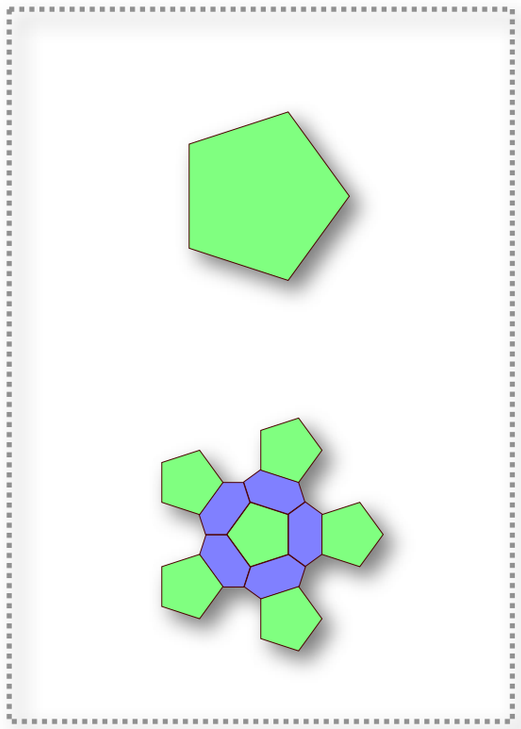
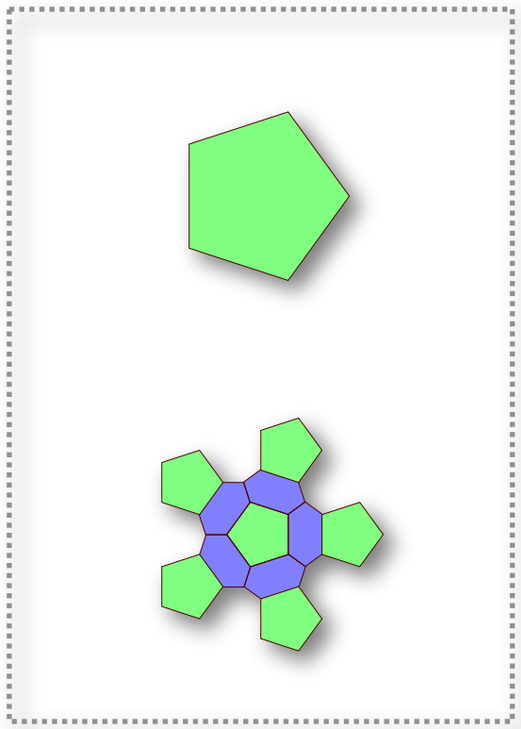
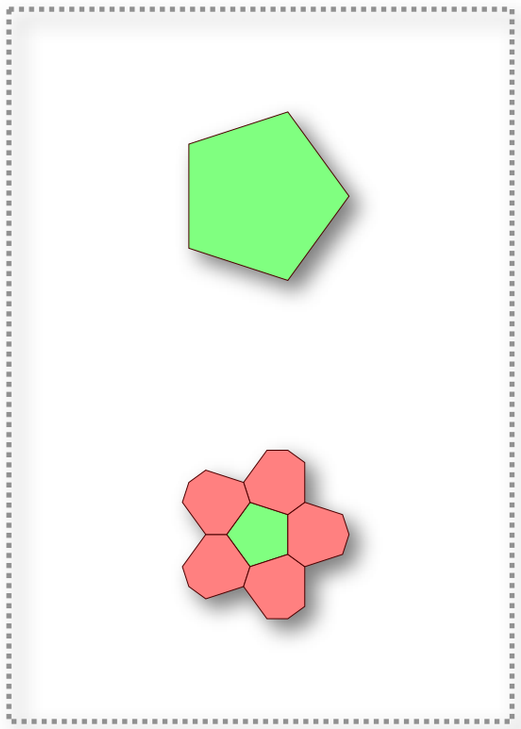
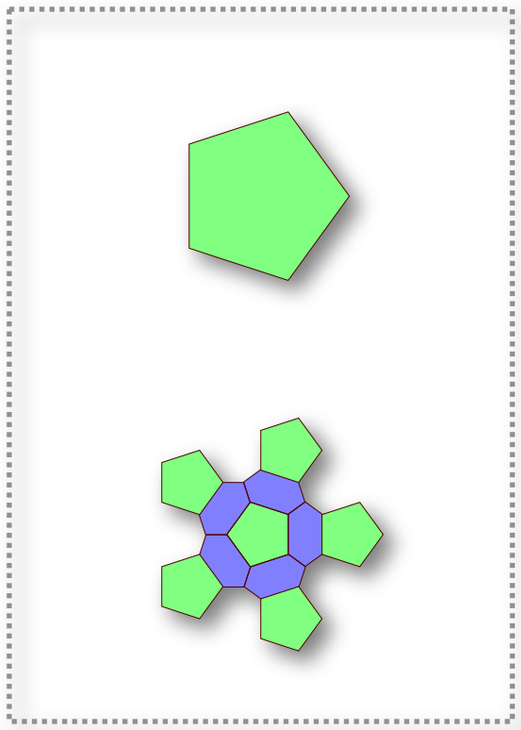
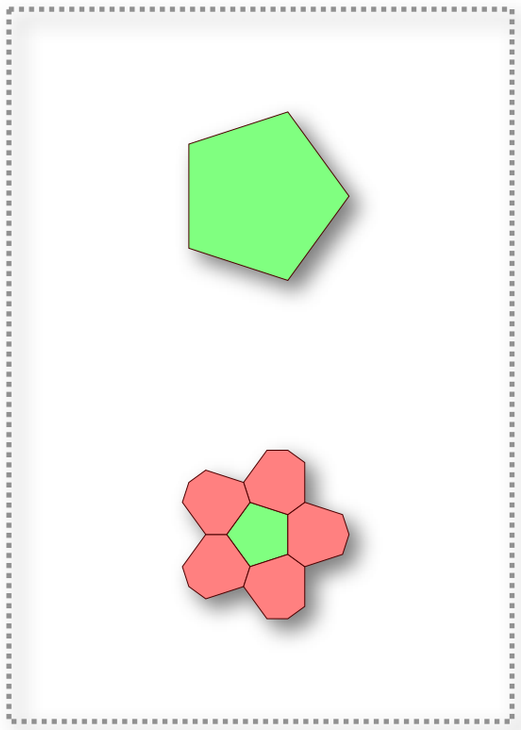
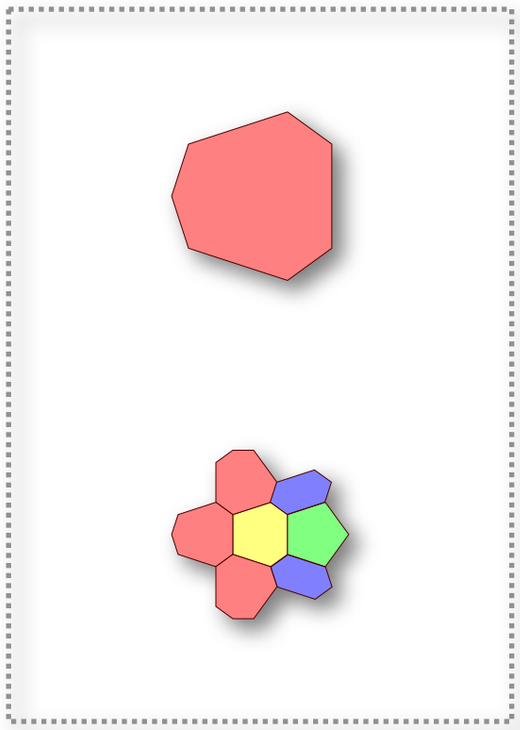


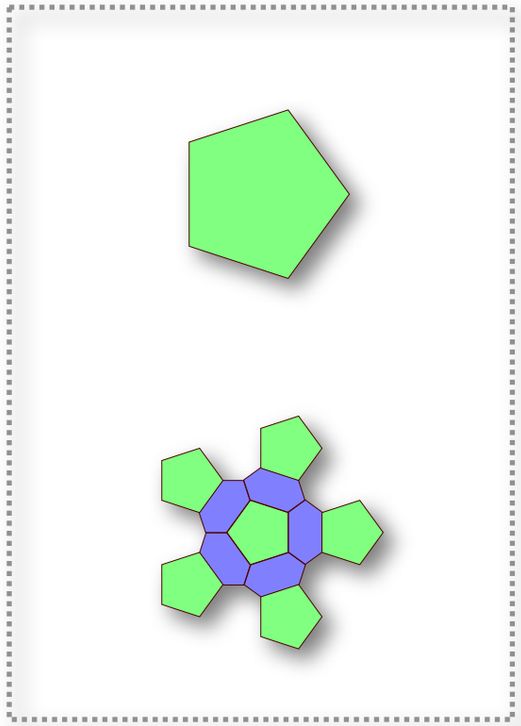


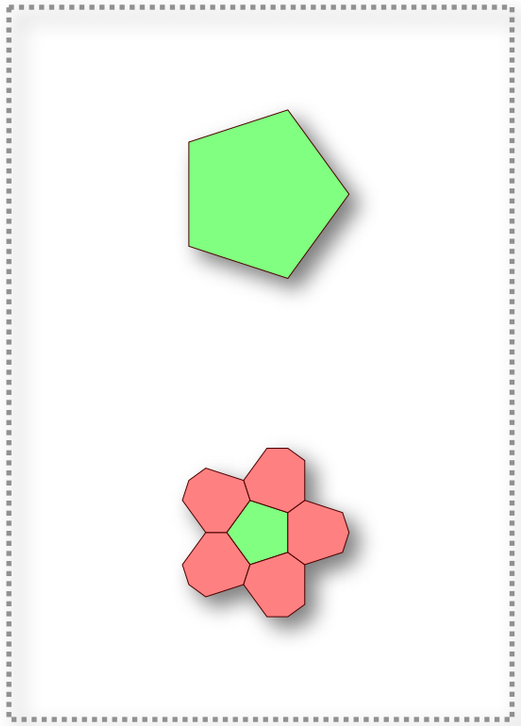


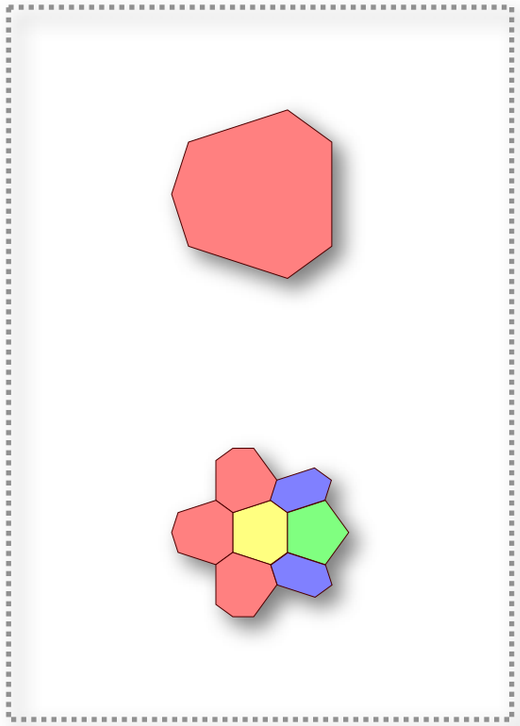


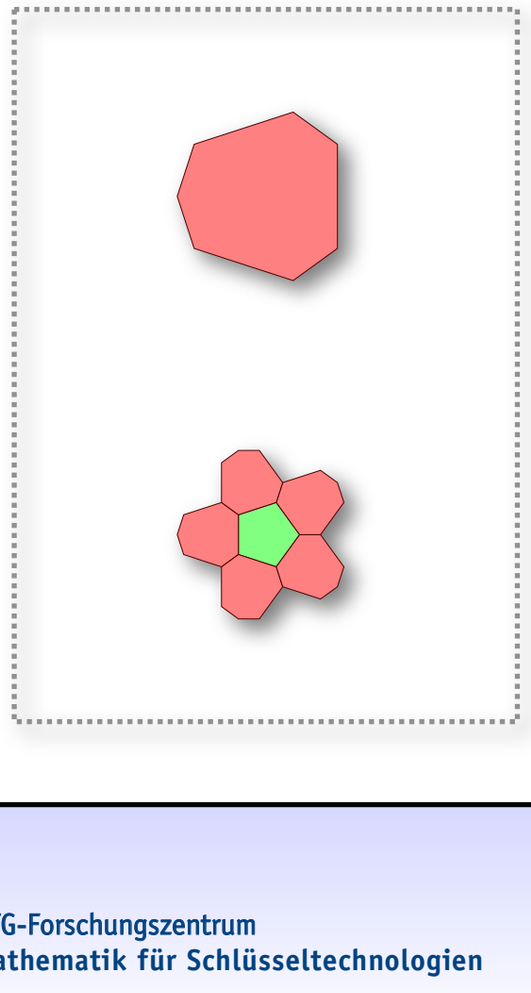


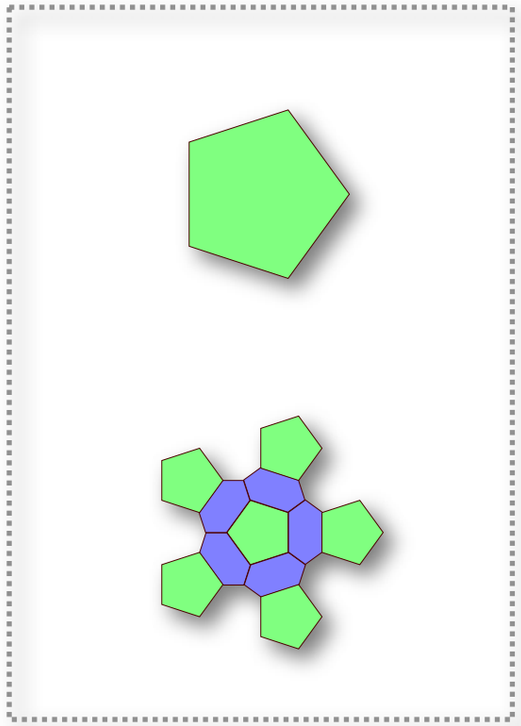


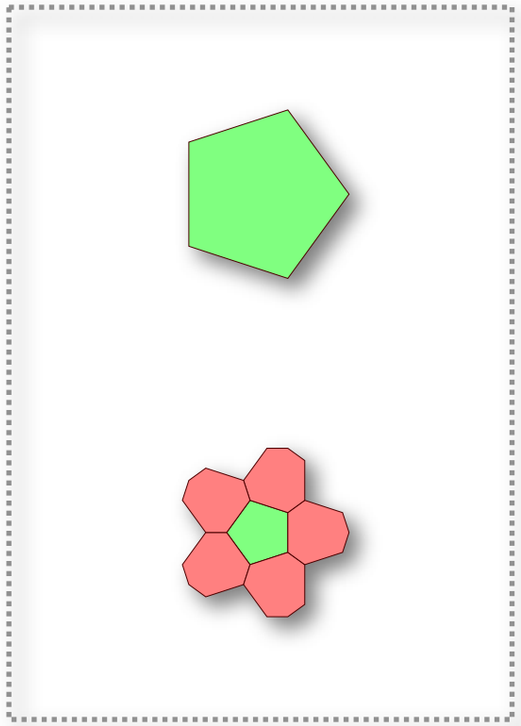


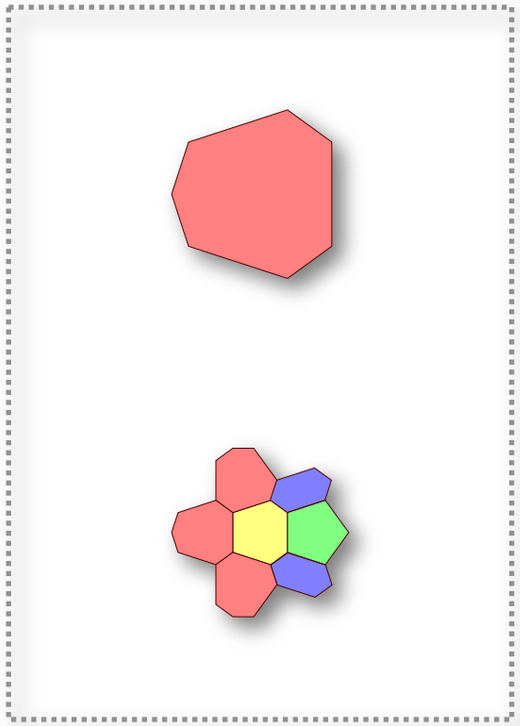


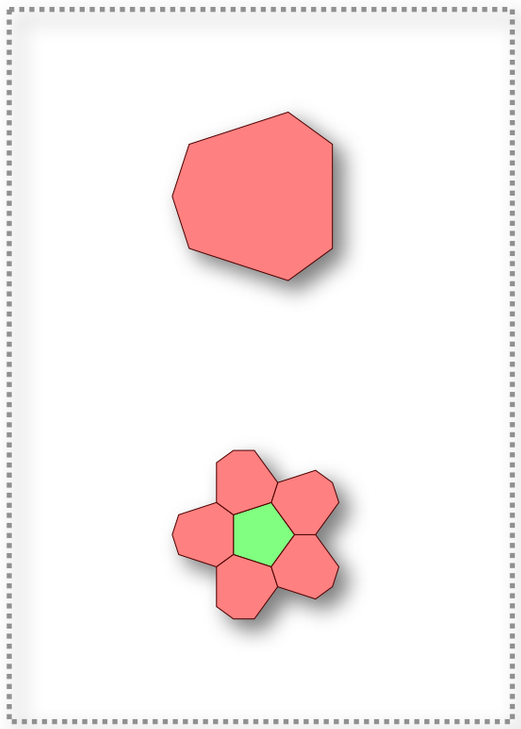


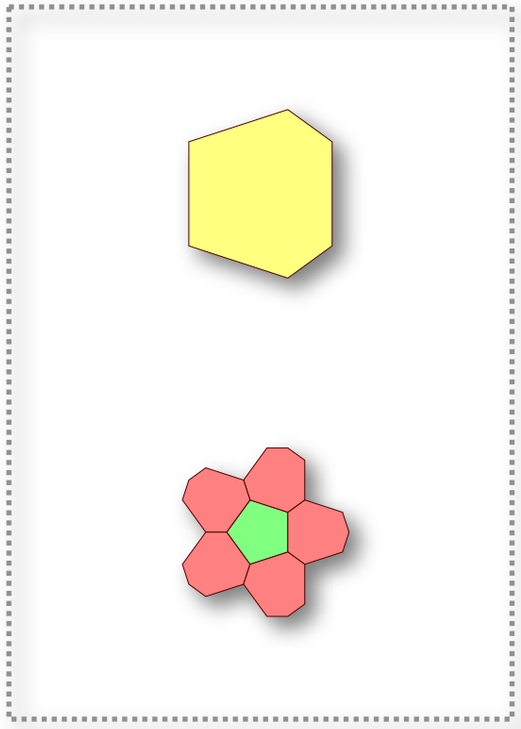


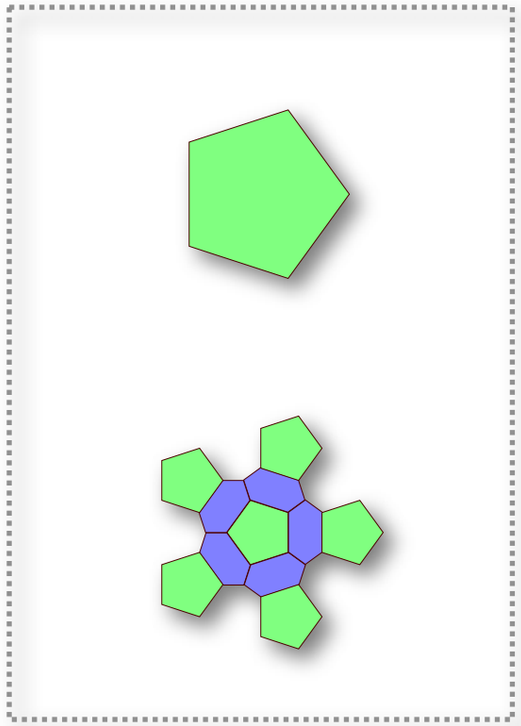


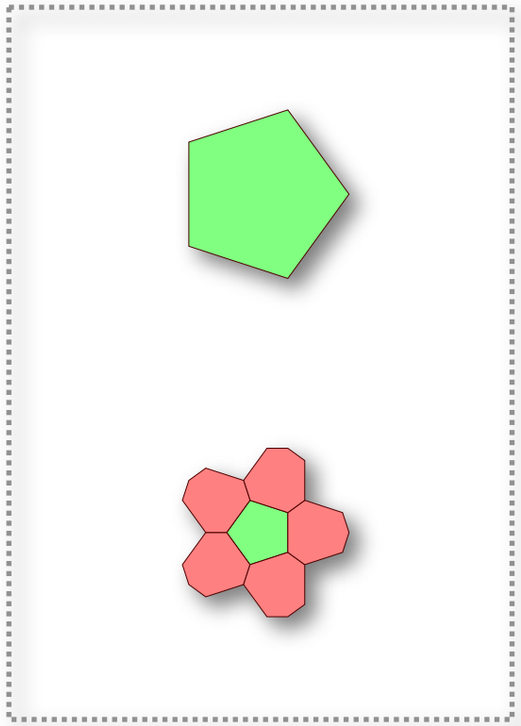


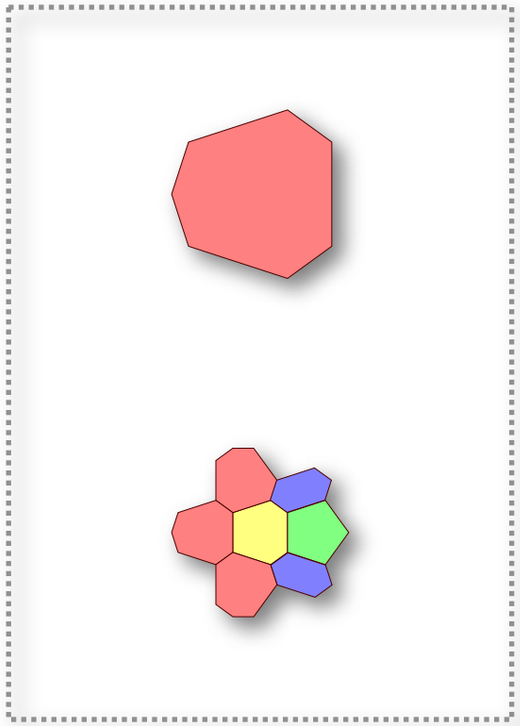


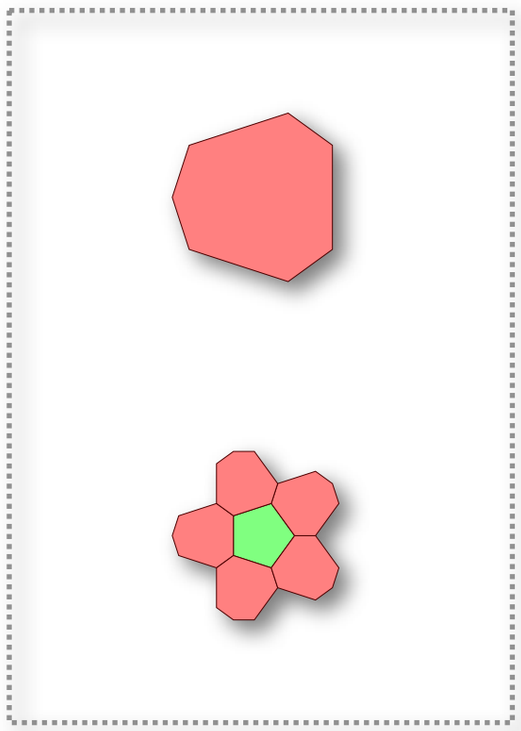


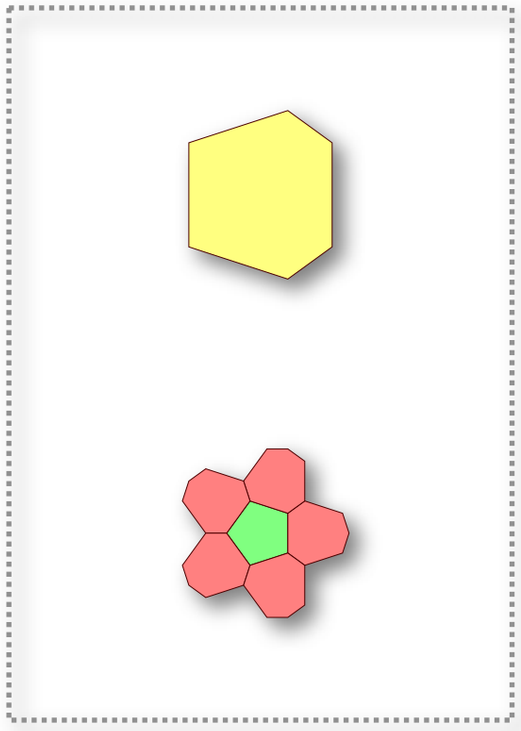


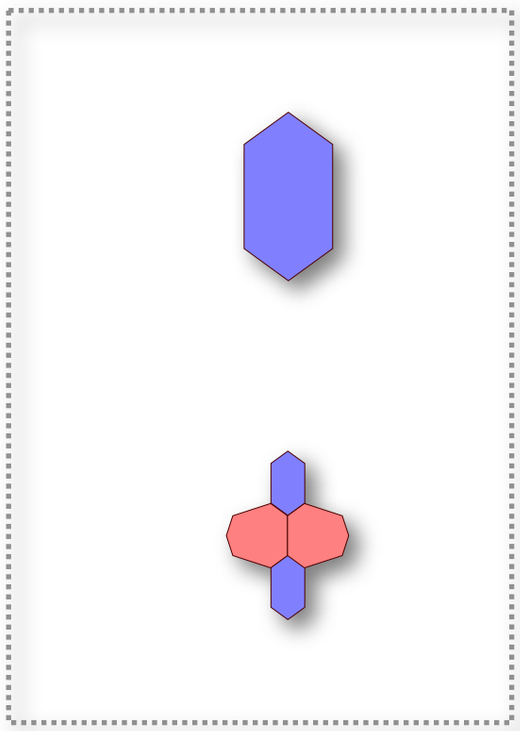


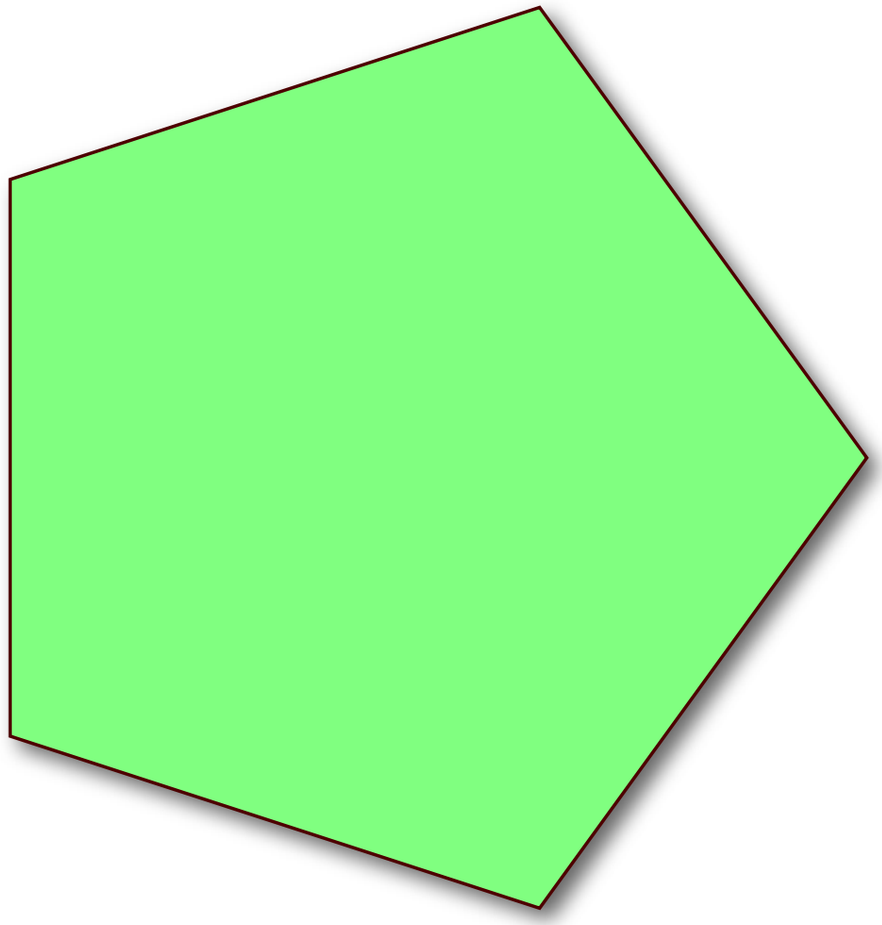


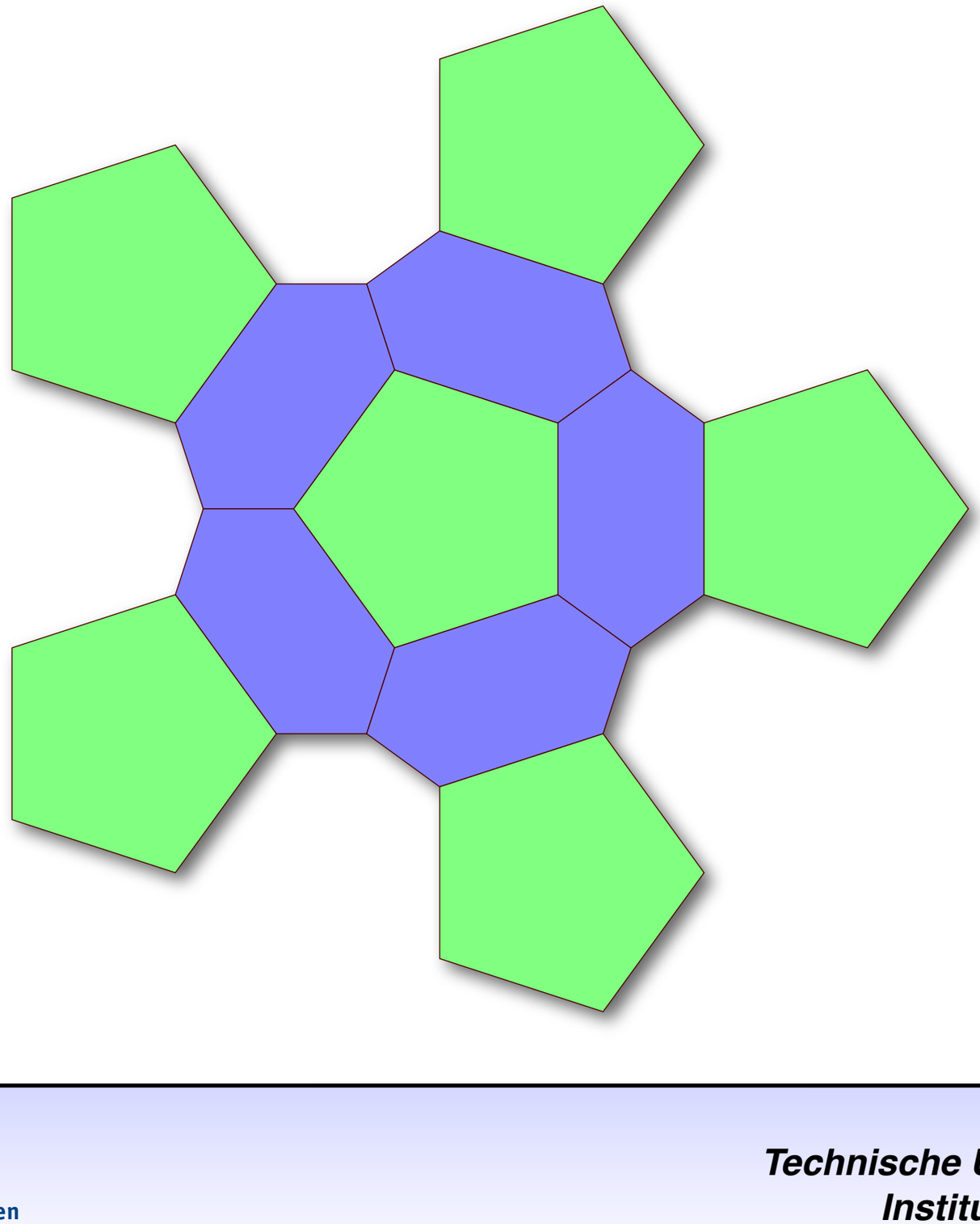


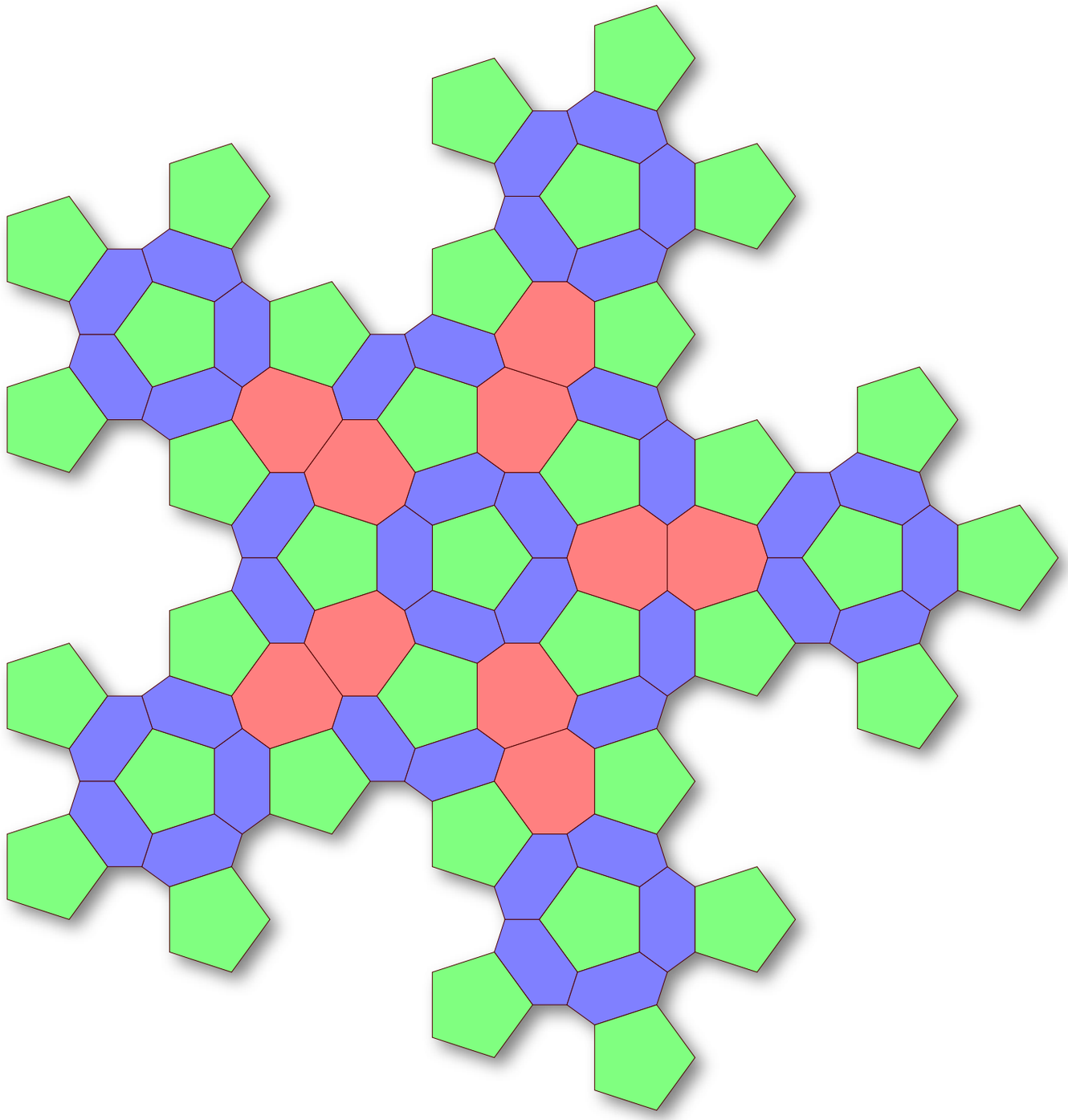






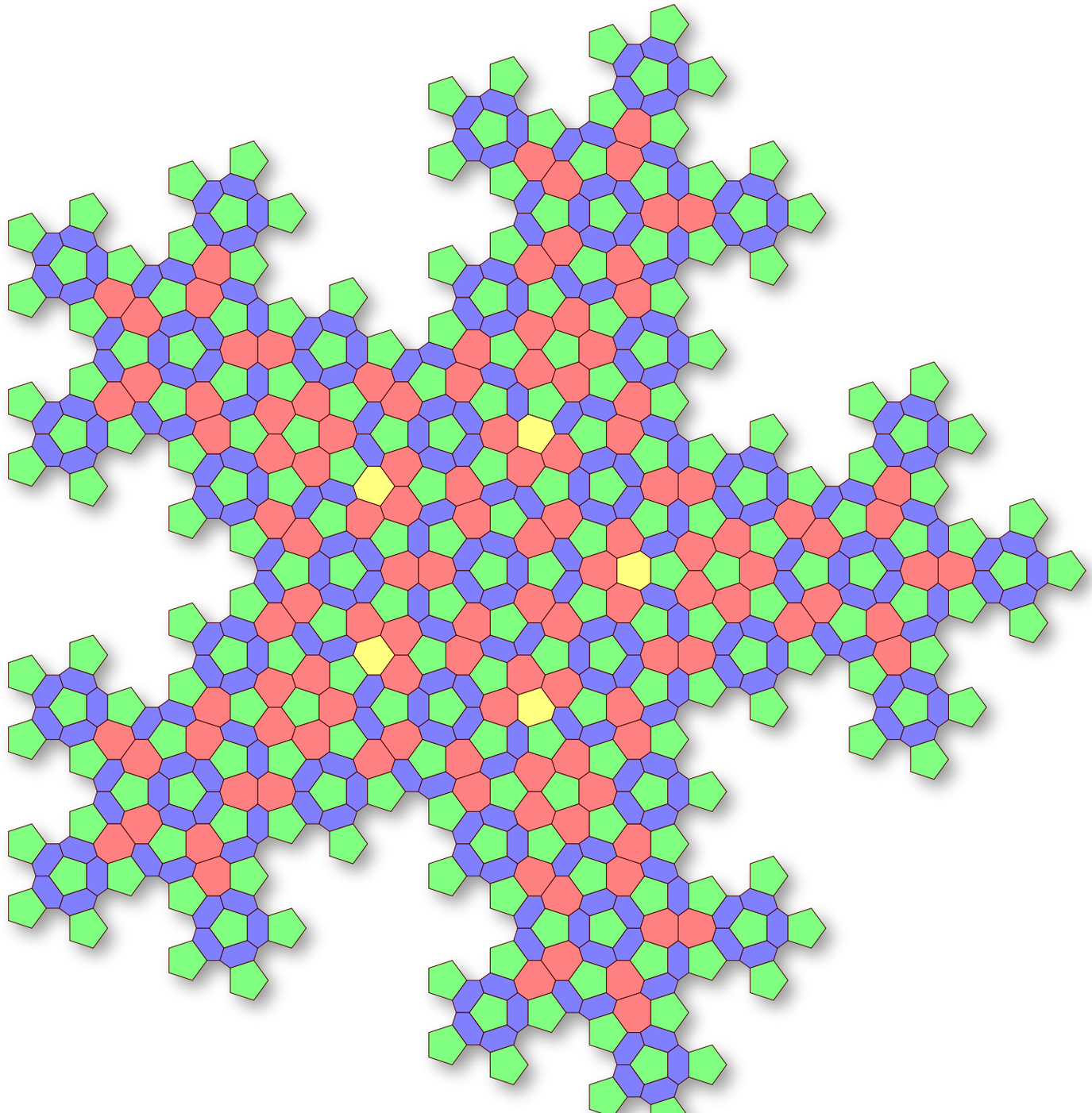






Berlin Alexanderplatz

Gestaltungswettbewerb

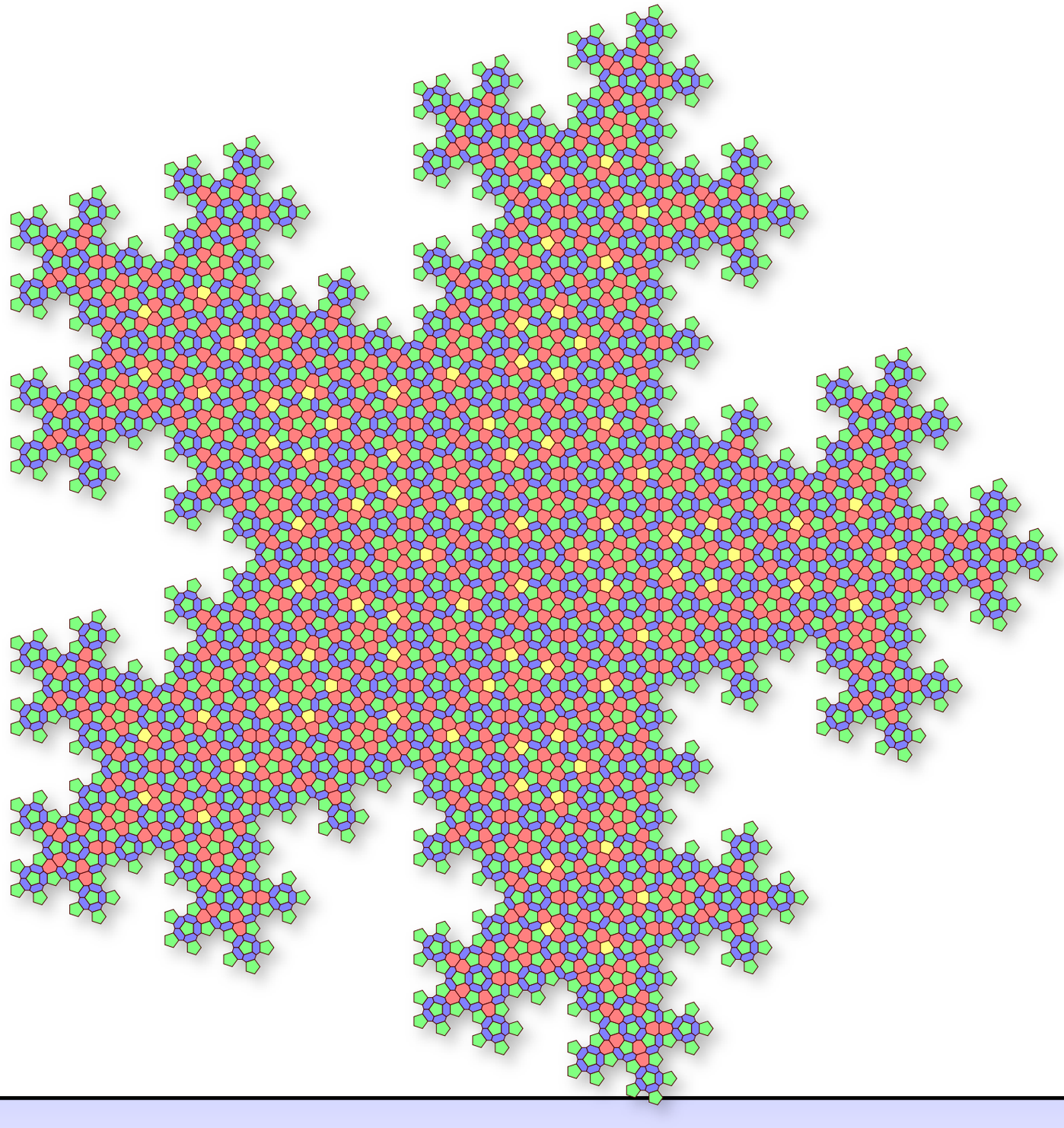


DFG-Forschungszentrum
Mathematik für Schlüsseltechnologien

Technische Universität Berlin
Institut für Mathematik

Berlin Alexanderplatz

Gestaltungswettbewerb



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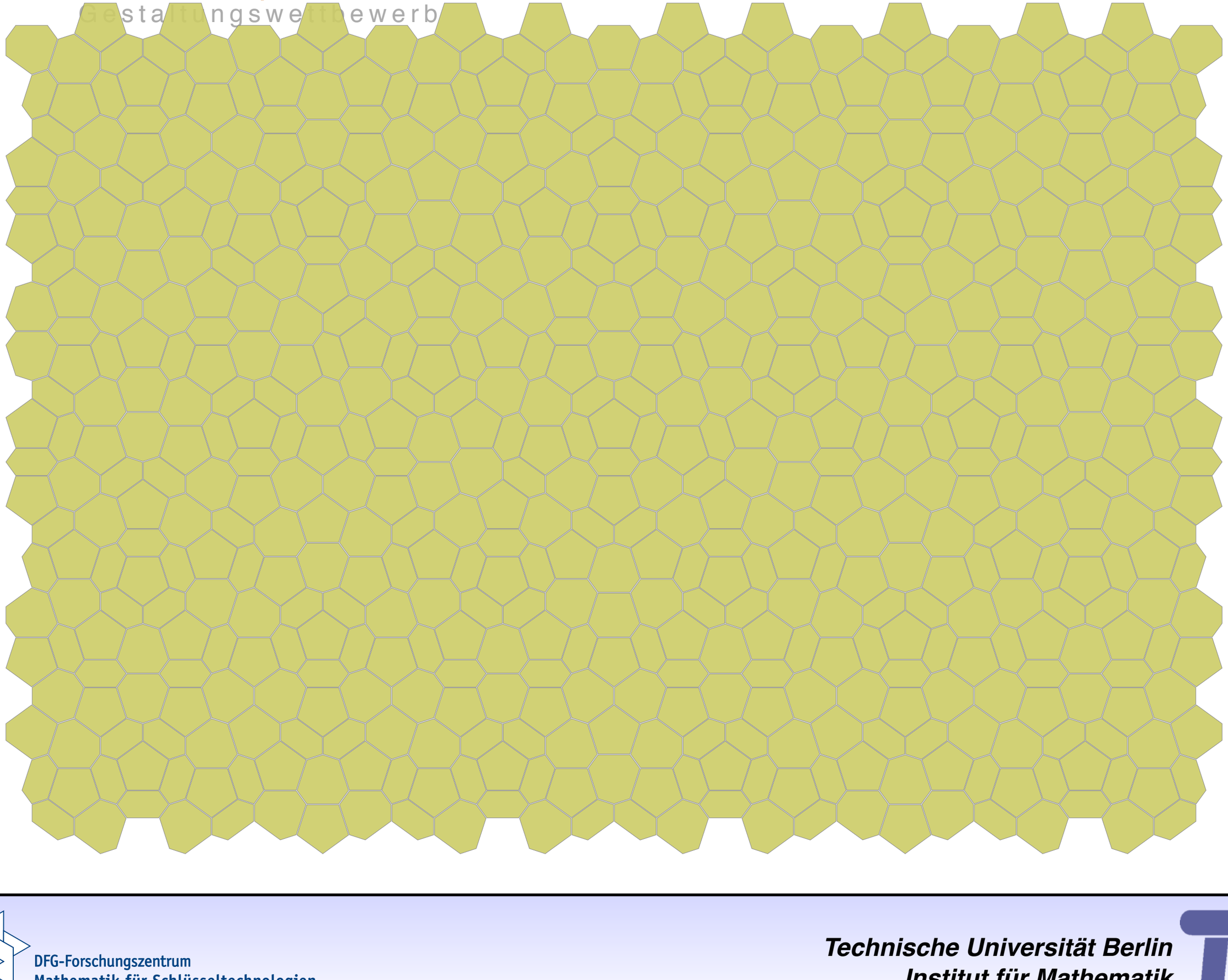


Indra's Pearls

- http://cinderella.de/indra

Part of

- http://mathe-vital.de

A wiki-based collection of high-quality OERs (Open Educational Resources) that complement lectures and books

Wikis

- Everybody knows Wikipedia
- All learning activities seem to end up in Wikipedia – good or bad?
- Web 2.0: Everybody can join – not really...
- New Paradigm: "Open"
- Necessary for us:

  Editorial work and **recognition of authors**

Madipedia.de

German initiative in mathematics education accepting modern learning

Madipedia will become the central point of information about mathematics education research:

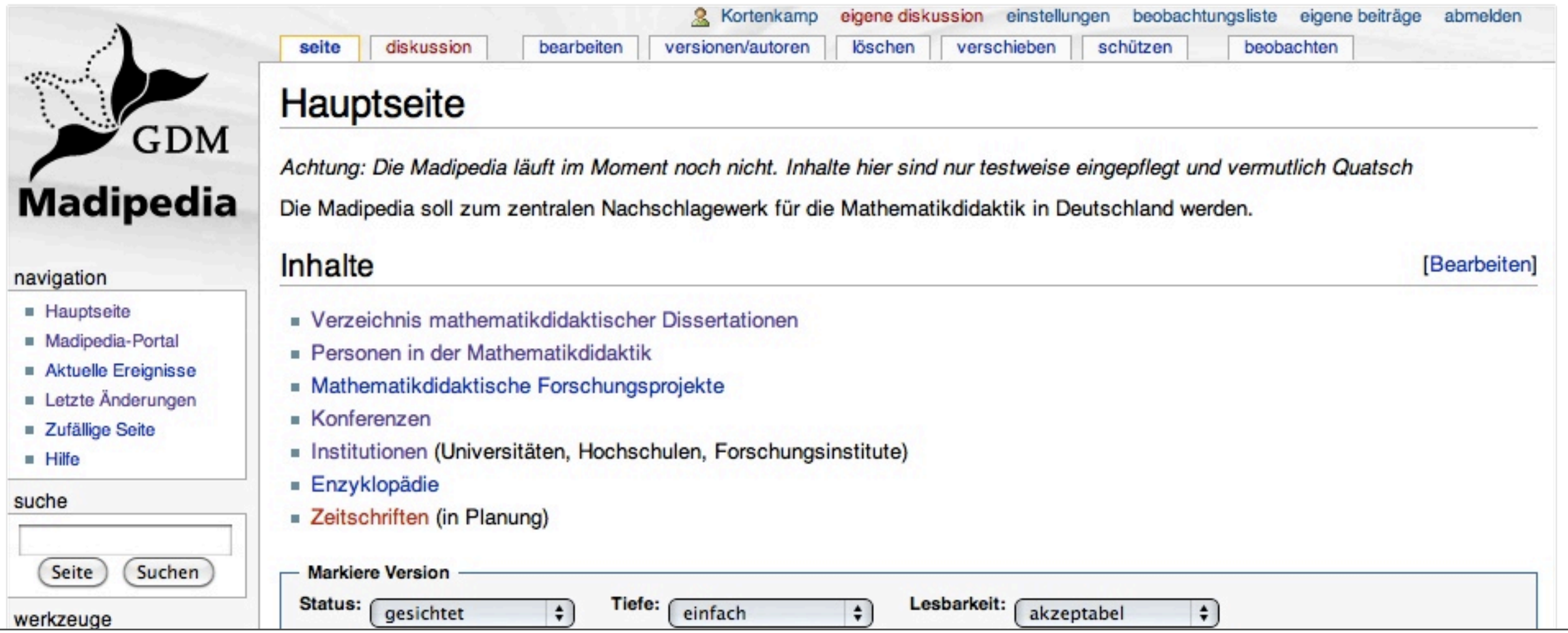
- Catalog of Ph.D. theses
- Persons in Mathematics Education
- Overview over Conferences and Projects
- Encyclopedia
- ...

Madipedia.de

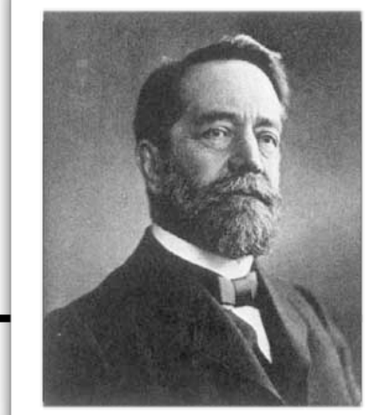
GDM

Goal: Easy access to all information in math education for better research of everybody!

Currently in beta-testing and fine-tuning of concepts & software.



Klein-Wiki (Madipedia.org?)



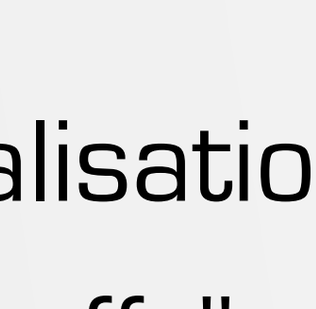
- Should be orthogonal to the book's topics
- High-Quality and reviewed content
- Citation-save source

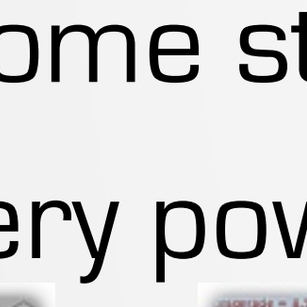
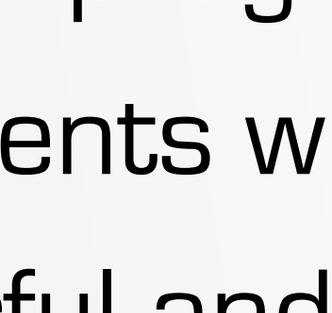
- Authors are visible and recognized
- Interactive content
- Editorial board needed
- Mathe-Vital.de as a basis?

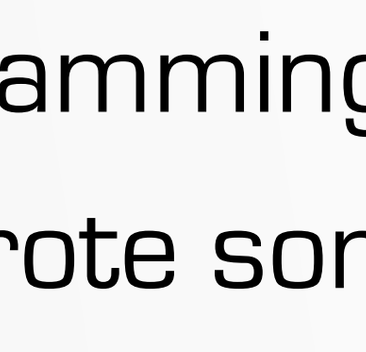
How to create content

- Formalisation through programming
- Banchoff: "some students wrote some code"
- There are very powerful and easy tools





Built into Cinderella: CindyScript

Functional Programming Language:

- non-typed, list-supporting

```
numbers = 1..100; i = 0;
odd numbers = apply(numbers,2*#+1);
```

- definitions

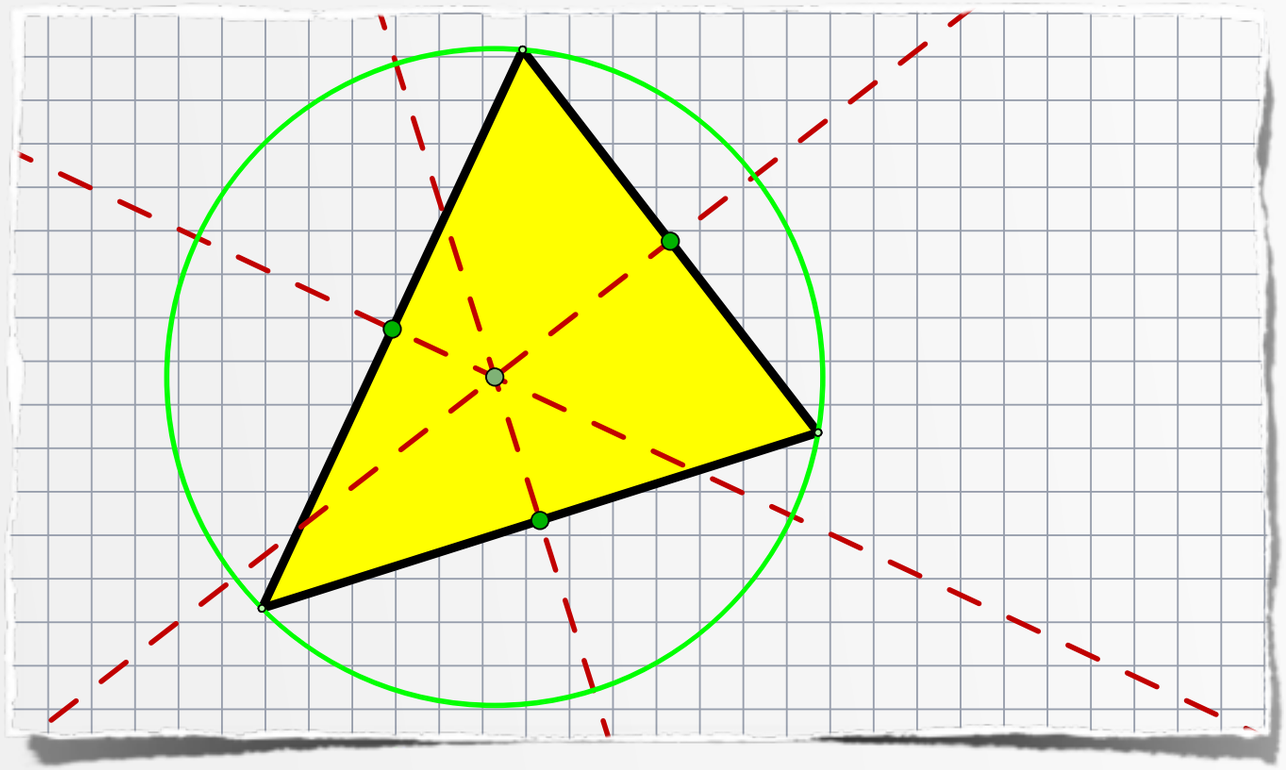
```
divisors(x) := select(1..x,mod(x,#)==0);
prime(p) := (length(divisors(p))==2);
```

- loops, drawing, ...

```
odd primes = select(odd numbers, prime(#));
forall(odd primes, draw([i,#]); i=i+1);
```

Teaching?

- Is it really necessary to have a programming language inside a teaching tool?



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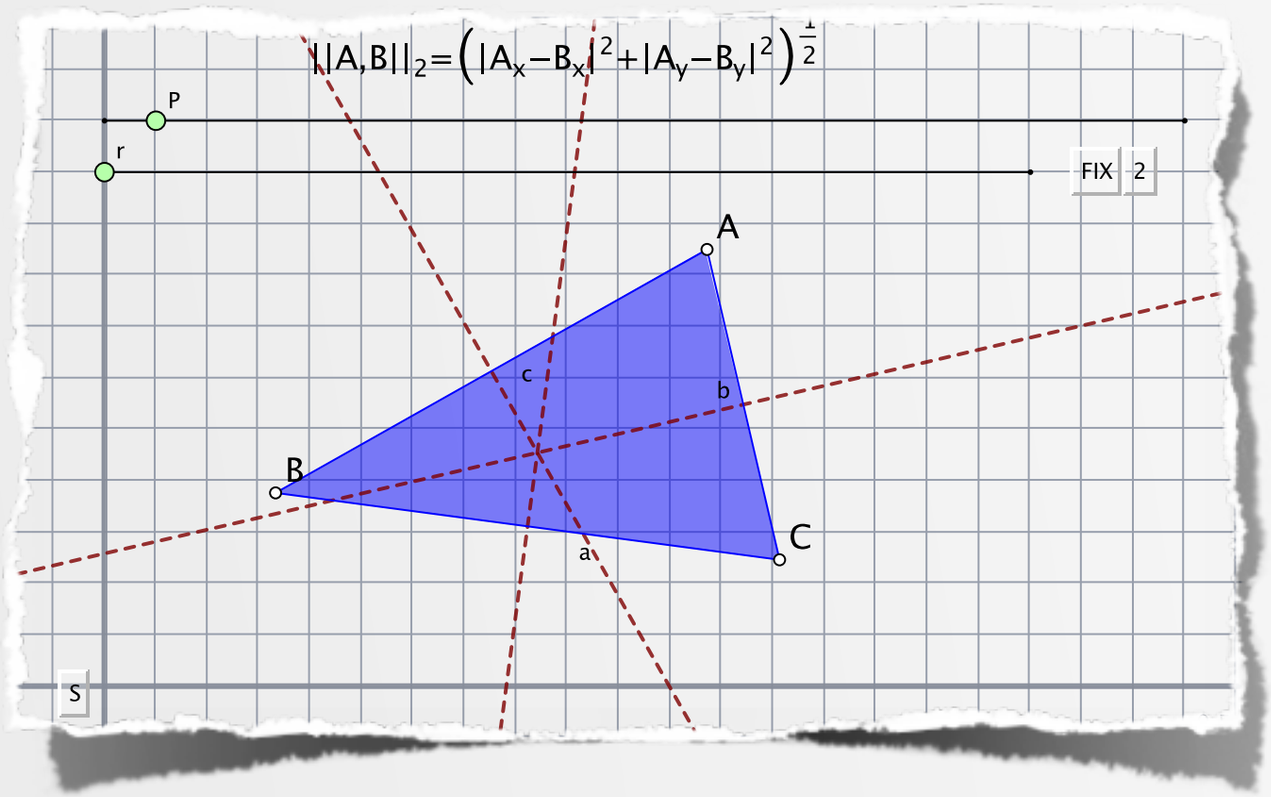
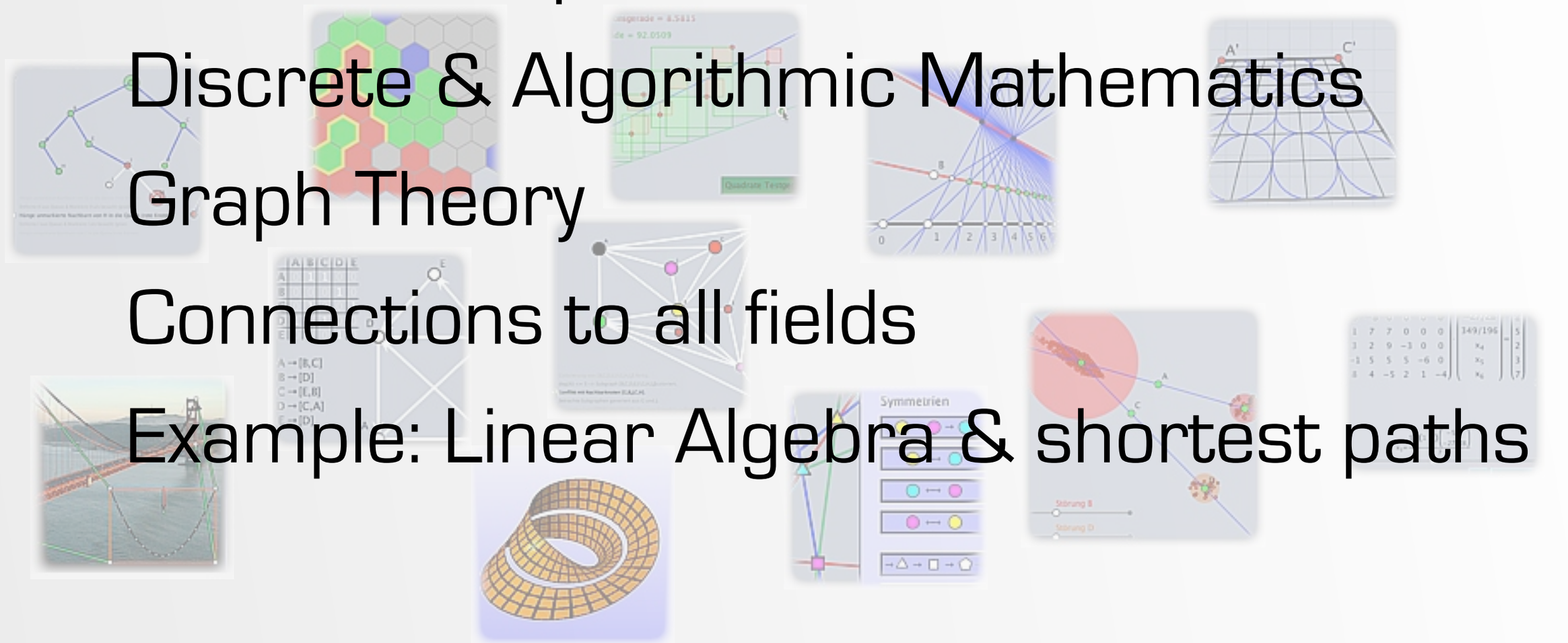


Yes: Because there is more than points, lines and circles in life!

Mathematical Processes

- **Change** & Analyze
- **Create** & Design
- **Decide** & Compare



http://twitter.com/ukor

Cheap GPS. (from: xkcd.com)

http://mathe-vital.de **http://kortenkamps.net**

http://cinderella.de **http://madipedia.de**

Moore's Law

(Wrong, but more spectacular version)

Computing power doubles every 18 months.

Or: Every two years everything becomes

twice as good

1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
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2001
2002
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2004
2005
2006
2007
2008
2009

	Harddisk
1980	
1981	
1982	
1983	
1984	
1985	
1986	
1987	
1988	
1989	
1990	60.000.000
1991	
1992	
1993	
1994	
1995	
1995	
1996	
1997	
1998	
1999	
2000	60.000.000.000
2001	
2002	120.000.000.000
2003	
2004	
2005	
2006	
2007	320.000.000.000
2008	
2009	1.000.000.000.000

	Harddisk	iPod
1980		
1981		
1982		
1983		
1984		
1985		
1986		
1987		
1988		
1989		
1990	60.000.000	
1991		
1992		
1993		
1994		
1995		
1996		
1997		
1998		
1999		
2000	60.000.000.000	
2001		
2002	120.000.000.000	5.000.000.000
2003		
2004		
2005		
2006		
2007	320.000.000.000	16.000.000.000
2008		
2009	1.000.000.000.000	32.000.000.000

	Harddisk	iPod	MHz
1980			
1981			
1982			
1983			4
1984			
1985			
1986			
1987			
1988			
1989			
1990	60.000.000		25
1991			
1992			
1993			
1994			
1995			
1996			
1997			
1998			
1999			
2000	60.000.000.000		450
2001			
2002	120.000.000.000	5.000.000.000	
2003			800
2004			
2005			1.670
2006			
2007	320.000.000.000	16.000.000.000	3.660
2008			
2009	1.000.000.000.000	32.000.000.000	4.800

	Harddisk	iPod	MHz	Memory
1980				
1981				
1982				
1983			4	16.384
1984				
1985				
1986				
1987				
1988				
1989				
1990	60.000.000		25	1.048.576
1991				
1992				
1993				
1994				
1995				
1996				
1997				
1998				
1999				
2000	60.000.000.000		450	536.870.912
2001				
2002	120.000.000.000	5.000.000.000		
2003			800	1.073.741.824
2004				
2005			1.670	
2006				
2007	320.000.000.000	16.000.000.000	3.660	
2008				3.221.225.472
2009	1.000.000.000.000	32.000.000.000	4.800	

	Harddisk	iPod	MHz	Memory	E-Mail
1980					
1981					
1982					
1983			4	16.384	
1984					
1985					
1986					
1987					
1988					
1989					
1990	60.000.000		25	1.048.576	
1991					
1992					
1993					
1994					
1995					432
1996					465
1997					400
1998					868
1999					1.472
2000	60.000.000.000		450	536.870.912	1.364
2001					2.007
2002	120.000.000.000	5.000.000.000			4.667
2003			800	1.073.741.824	2.762
2004					4.250
2005			1.670		8.621
2006					13.271
2007	320.000.000.000	16.000.000.000	3.660		13.996
2008				3.221.225.472	14.519
2009	1.000.000.000.000	32.000.000.000	4.800		15.135

On a log scale it's correct